Mathematics overview: Stage 6*

Unit	Hours	KNOWLEDGE
Numbers and the number system	6	• identify the value of each digit in numbers given to three decimal places and multiply and divide numbers by 10, 100 and 1000 giving answers up to three decimal places (ALSO • 7*) • read, write, order and compare numbers up to 10 000 000 and determine the value of each digit • use negative numbers in context, and calculate intervals across zero (including addition and subtraction of negative numbers) • identify common factors, common multiples and prime numbers • use positive integer powers and associated real roots (square, cube and higher), recognise powers of 2,3,4,5 (ALSO 7* AND NEW)
Calculating	6	
Calculating: division	6	
Visualising and constructing	6	• perform mental calculations, including with mixed operations and large numbers • solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why (also 7*) • multiply multi-digit numbers up to 4 digits by a two-digit whole number using the formal written method of long multiplication • solve problems involving addition, subtraction, multiplication and division (ALSO 7*) • apply the four operations, including formal written methods, to integers and decimals. (NEW) • use the symbols =, ≠, <, >, ≤, ≥ (ALSO 7*)
Investigating properties of shapes	9	• divide numbers up to 4 digits by a two-digit whole number using the formal written method of long division; interpret remainders as whole number remainders, fractions, or by rounding, as appropriate for the context • use written division methods in cases where the answer has up to two decimal places • use their knowledge of the order of operations to carry out calculations involving the four operations • recognise, describe and build simple 3-D shapes, including making nets
Algebraic proficiency: tinkering	9	• use conventional terms and notations: points, lines, vertices, edges, planes, parallel lines, perpendicular lines, right angles, polygons, regular polygons and polygons with reflection and/or rotation symmetries (ALSO 7*) • use the standard conventions for labelling and referring to the sides and angles of triangles (ALSO 7*)
Exploring fractions, decimals and percentages	9	• compare and classify geometric shapes based on their properties and sizes and find unknown angles in any triangles, quadrilaterals, and regular polygons (ALSO 7*) • draw 2-D shapes using given dimensions and angles (NEW) • draw diagrams from written descriptions (NEW, ALSO 7*) • illustrate and name parts of circles, including radius, diameter and circumference and know that the diameter is twice the radius • identify properties of the faces, surfaces, edges and vertices of: cubes, cuboids, prisms, cylinders, pyramids, cones and spheres
Proportional reasoning	6	• understand and use the concepts and vocabulary of expressions, equations, formulae, terms and factors (ALSO 7*) • use and interpret algebraic notation, including: ab in place of a × b, 3y in place of y + y + y and 3 × y, a² in place of a × a, a³ in place of a × a × a, a/b in place of a ÷ b, brackets (ALSO 7*) • simplify and manipulate algebraic expressions by collecting like terms and multiplying a single term over a bracket (ALSO 7*)
Pattern sniffing	3	• substitute numerical values into formulae and expressions • use conventional notation for priority of operations, including brackets
Measuring space	3	• use common factors to simplify fractions; use common multiples to express fractions in the same denomination • compare and order fractions, including fractions > 1
Investigating angles	3	• associate a fraction with division and calculate decimal fraction equivalents [for example, 0.375] for a simple fraction [for example, 3/8] (ALSO 7*) • recall and use equivalences between simple fractions, decimals and percentages, including in different contexts (ALSO 7*) • solve problems involving the relative sizes of two quantities where missing values can be found by using integer multiplication and division facts
Calculating fractions, decimals and percentages	9	• solve problems involving similar shapes where the scale factor is known or can be found • solve problems involving unequal sharing and grouping using knowledge of fractions and multiples • use ratio notation, including reduction to simplest form (ALSO 7*) • divide a given quantity into two parts in a given part:part or part:whole ratio (ALSO 7*)

[Solving equations and inequalities](#)	3	• generate and describe linear number sequences • generate terms of a sequence from a term-to-term rule • use, read, write and convert between standard units, converting measurements of length, mass, volume and time from a smaller unit of measure to a larger unit, and vice versa, using decimal notation to up to three decimal places (ALSO 7*) • convert between miles and kilometres (NEW, ALSO 7*) • use standard units of mass, length, time, money and other measures using decimal quantities where appropriate (ALSO 7*) • recognise angles where they meet at a point, are on a straight line, or are vertically opposite, and find missing angles • apply the properties of angles at a point, angles at a point on a straight line, vertically opposite angles (ALSO 7*) • add and subtract fractions with different denominators and mixed numbers, using the concept of equivalent fractions (ALSO 7*) • multiply simple pairs of proper fractions, writing the answer in its simplest form [for example, 1/4 × 1/2 = 1/8] (ALSO 7*) • divide proper fractions by whole numbers [for example, 1/3 ÷ 2 = 1/6] (also 7*) • multiply one-digit numbers with up to two decimal places by whole numbers (ALSO 7*) • solve problems involving the calculation of percentages [for example, of measures, and such as 15% of 360] and the use of percentages for comparison • enumerate possibilities of combinations of two variables • express missing number problems algebraically • find pairs of numbers that satisfy an equation with two unknowns • recognise that shapes with the same areas can have different perimeters and vice versa • calculate the area of parallelograms and triangles • calculate, estimate and compare volume of cubes and cuboids using standard units, including cubic centimetres (cm^3) and cubic metres (m^3), and extending to other units [for example, mm^3 and km^3] • recognise when it is possible to use formulae for area and volume of shape • solve problems involving the calculation and conversion of units of measure, using decimal notation up to three decimal places where appropriate • calculate perimeters of 2D shapes (ALSO 7*) • calculate surface area of cuboids • solve problems which require answers to be rounded to specified degrees of accuracy • use estimation to check answers to calculations and determine, in the context of a problem, an appropriate degree of accuracy • round any whole number to a required degree of accuracy • describe positions on the full coordinate grid (all four quadrants) • work with co-ordinates in all four quadrants (NEW) • draw and translate simple shapes on the coordinate plane, and reflect them in the axes • solve geometric problems on coordinate axes • describe translations as 2D vectors (ALSO 7*) • interpret and construct pie charts and line graphs and use these to solve problems • calculate and interpret the mean as an average • interpret, analyse and compare the distributions of data sets from univariate empirical distributions through appropriate measures of central tendency (median, mean and mode) and spread (range) (ALSO 7*) • Relate relative expected frequencies to theoretical probability, using appropriate language and the 0-1 probability scale (NEW) • apply the property that the probabilities of an exhaustive set of outcomes sum to one
[Calculating space](#)	9	
[Checking, approximating and estimating](#)	3	
[Mathematical movement](#)	3	
[Presentation of data](#)	3	
[Measuring data](#)	3	
[Understanding Risk 1](#)	3	

Numbers and the number system	_Link To Stage 7*_	_6 hours_

KNOWLEDGE   **The Big Picture**: Number and Place Value progression map

- identify the value of each digit in numbers given to three decimal places and multiply and divide numbers by 10, 100 and 1000 giving answers up to three decimal places (ALSO
- 7*)
- read, write, order and compare numbers up to 10 000 000 and determine the value of each digit
- use negative numbers in context, and calculate intervals across zero (including addition and subtraction of negative numbers)
- identify common factors, common multiples and prime numbers
- use positive integer powers and associated real roots (square, cube and higher), recognise powers of 2,3,4,5 (ALSO 7* AND NEW)

SKILLS	Suggested resources
- Understand place value in numbers with up to three decimal places - Multiply whole numbers by 10 (100, 1000) - Divide whole numbers by 10 (100, 1000) when the answer is a whole number - Multiply (divide) numbers with up to three decimal places by 10 (100, 1000) - Understand (order, write, read) place value in numbers with up to eight digits - Understand and use negative numbers when working with temperature - Understand and use negative numbers when working in other contexts - Know the meaning of a common multiple (factor) of two numbers - Identify common multiples (factors) of two numbers Know how to test if a number up to 120 is prime	KM: Maths to Infinity: Directed numbers KM: Extend the idea of Eratosthenes' sieve to a 12 by 12 grid KM: Exploring primes activities: Artistic Eratosthenes sieve KM: Use Powers of ten to demonstrate connections. NRICH: Factor-multiple chains NRICH: The Moons of Vuvv NRICH: Round and round the circle NRICH: Counting cogs **Learning review** www.diagnosticquestions.com

Prerequisites	Mathematical language	Agreed Common Teaching Approaches
- Understand and use place value in numbers with up to seven digits - Multiply and divide whole numbers by 10, 100, 1000 - Multiply and divide numbers with one decimal place by 10, 100, 1000 - Know the meaning of 'factor' and 'multiple' and 'prime' **Bring on the Maths⁺: Moving on up!** Number and Place Value: v1 Solving problems: v3	Place value Digit Negative number (Common) multiple (Common) factor Divisible Prime number, Composite number	Zero is neither positive nor negative. When multiplying and dividing by powers of ten, the decimal point is fixed and it is the digits that move. Ensure that pupils can deal with large numbers that include zeros in the HTh and/or H column (e.g. 43 006 619) NCETM: Glossary **Common approaches** _The following definition of a prime number should be used in order to minimise confusion about 1: A prime number is a number with exactly two factors._ _Every classroom has a set of number classification posters, a place chart and a negative number washing line on the wall._

Reasoning opportunities and probing questions	Cross Curicular Links	Possible misconceptions
- Convince me that 109 is a prime number - Jenny writes 2.54 × 10 = 25.4. Kenny writes 2.54 × 10 = 25.40. Who do you agree with? Explain why. - Look at this number (24 054 028). Show me another number (with 4, 5, 6, 7 digits) that includes a 5 with the same value. And another. And another …		- Some pupils can confuse the language of large (and small) numbers since the prefix 'milli- means 'one thousandth' (meaning that there are 1000 millimetres in a metre for example) while one million is actually a thousand thousand. - Some pupils may not realise that degrees (°) and degrees Celsius (°C) are two different and distinct units of measurement - Some pupils may think that 1 is a prime number

KNOWLDGE   **The Big Picture**: Calculation progression map

- **perform mental calculations, including with mixed operations and large numbers**
- **solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why (also 7*)**
- **multiply multi-digit numbers up to 4 digits by a two-digit whole number using the formal written method of long multiplication**
- **solve problems involving addition, subtraction, multiplication and division (ALSO 7*)**
- **apply the four operations, including formal written methods, to integers and decimals**. (NEW)
- **use the symbols =, ≠, <, >, ≤, ≥ (ALSO 7*)**

SKILLS	Suggested resources
• Combine addition and subtraction when multiplying mentally • Multiply a two-digit number by a single-digit number mentally • Add a three-digit number to a two-digit number mentally (when bridging of hundreds is required) • Multiply a four-digit number by a two-digit number using long multiplication • Identify when addition, subtraction or multiplication is needed as part of solving multi-step problems • Explain why addition or subtraction is needed at any point when solving multi-step problems • Solve multi-step problems involving addition, subtraction and/or multiplication • Know that addition and subtraction have equal priority • Know that multiplication and division have equal priority Know that multiplication and division take priority over addition and subtraction	KM: Long multiplication template KM: Maximise, minimise. Adapt ideas to fit learning intentions. KM: Maths to Infinity: Complements KM: Maths to Infinity: Multiplying and dividing NRICH: Become Maths detectives NRICH: Exploring number patterns you make NRICH: Reach 100 **Learning review** • www.diagnosticquestions.com

Prerequisites	Mathematical language	Agreed Common Teaching Approaches
• Recall multiplication facts for multiplication tables up to 12 × 12 • Recall division facts for multiplication tables up to 12 × 12 • Understand the commutativity of multiplication and addition • Multiply a three-digit number by a two-digit number using short multiplication • Use column addition and subtraction for numbers with more than four digits **Bring on the Maths⁺: Moving on up!** Calculating: v1 Solving problems: v1	Addition Subtraction Sum, Total Difference, Minus, Less Column addition Column subtraction Operation Multiply, Multiplication, Times, Product Commutative Factor Short multiplication Long multiplication Estimate	The grid method is promoted as a method that aids numerical understanding and later progresses to multiplying algebraic statements. Use a basic and a scientific calculator to work out 2 + 3 × 5. Why are the answers different? KM: Calculation overview NCETM: The Bar Model I: addition and subtraction NCETM: The Bar Model II: multiplication and division NCETM: Subtraction and multiplication NCETM: Multiplicative reasoning NCETM: Glossary **Common approaches** *To avoid confusion with language, all teachers use 'sum' to refer only to the result of an addition. Teachers say 'complete these calculations' instead of 'complete these sums'* *Long multiplication is promoted as the 'most efficient method'.* *If any acronym is promoted to help remember the order of operations, then BIDMAS is used to strengthen progression as the I stands for indices.*

Reasoning opportunities and probing questions	Cross Curricular Links	Possible misconceptions
• Find missing digits in otherwise completed long multiplication calculations • Convince me that 2472 × 12 = 29664 • Why have you chosen to add (subtract, multiply)?		• Some pupils may write statements such as 140 - 190 = 50 • When subtracting mentally some pupils may deal with columns separately and not combine correctly; e.g. 180 – 24: 180 – 20 = 160. Taking away 4 will leave 6. So the answer is 166. • The use of BIDMAS (or BODMAS) can imply that division takes priority over multiplication, and that addition takes priority over subtraction. This can result in incorrect calculations.

Calculating: division	Link To Stage 7*		_6 hours_

KNOWLDGE	**The Big Picture**: Calculation progression map

- **divide numbers up to 4 digits by a two-digit whole number using the formal written method of long division; interpret remainders as whole number remainders, fractions, or by rounding, as appropriate for the context**
- **use written division methods in cases where the answer has up to two decimal places**
- **use their knowledge of the order of operations to carry out calculations involving the four operations**

SKILLS	Suggested resources
- Use short division to divide a four-digit number by a one-digit number - Use short division to divide a three- (or four-) digit number by a two-digit number - Understand the method of long division - Use long division to find the remainder at each step of the division - Know how to write, and use, the remainder at each step of the division - Use long division to divide a three- (or four-) digit number by a two-digit number - Write the remainder to a division problem as a remainder - Write the remainder to a division problem as a fraction - Extend beyond the decimal point to write the remainder as a decimal - Identify when division is needed to solve a problem - Extract the correct information from a problem and set up a written division calculation Interpret a remainder when carrying out division	KM: Dividing (lots) KM: Maths to Infinity: Multiplying and dividing KM: Interactive long division NRICH: Interactive division NRICH Dicey Operations. Game 6. **Learning review** www.diagnosticquestions.com

Prerequisites	Mathematical language	Agreed Common Teaching Approaches
- Use knowledge of multiplication tables when dividing - Know how to use short division	Commutative Divide, Division, Divisible Divisor, Dividend, Quotient, Remainder Factor Short division Long division Remainder Operation Estimate **Notation** Remainders are often abbreviated to 'r' or 'rem'	Long division is a method of division that formalises how to find a remainder at each step. Short division is a compact method that uses pupils' abilities to find this remainder without the need for a formal written method. An alternative to both these methods is 'Galley division'. KM: Calculation overview NCETM: The Bar Model NCETM: Division NCETM: Glossary **Common approaches** _The use of long multiplication is promoted as the 'most efficient method'._ _Short division is promoted as the 'most efficient method'._ _When dealing with remainders in division problems, use the notation 'r'_

Reasoning opportunities and probing questions	Cross Curicular Links	Possible misconceptions
- Find missing digits in otherwise completed long / short division calculations - Show me a calculation that is connected to 147 × 26 = 3822. And another, and another … - Show me a division calculation that has no remainder. Now show me a division by a two-digit number that has no remainder. And now, a four-digit number divided by a two-digit number that has no remainder. And now, with a remainder of 3 …		- Some pupils may write statements such as 12 ÷ 132 = 11 - Formal written methods of addition, subtraction and multiplication work from right to left. Formal division works from left to right. - When using short division many pupils will at first struggle to deal correctly with any division where the divisor is greater than the first digit of the dividend; for example: $$0 \quad \textcolor{red}{10} \quad 7 \qquad r\,5$$ $$8 \,\big|\, 3 \quad {}^8 6 \quad {}^6 1$$ 3 ÷ 8 = 0 remainder 3, and so the 3 should be moved across. Instead, the 8 has been 'moved across' and therefore everything that follows has been correctly carried out based on an early misunderstanding.

Visualising and constructing	Link To Stage 7*	*6 hours*

KNOWLEDGE

The Big Picture: Properties of Shape progression map

- **recognise, describe and build simple 3-D shapes, including making nets**
- **use conventional terms and notations: points, lines, vertices, edges, planes, parallel lines, perpendicular lines, right angles, polygons, regular polygons and polygons with reflection and/or rotation symmetries (ALSO 7*)**
- **use the standard conventions for labelling and referring to the sides and angles of triangles (ALSO 7*)**

SKILLS	Suggested resources
- Use a protractor to measure and draw angles up to 180° - Use a protractor to measure and construct reflex angles - Use a ruler to draw lines to the nearest millimetre - Use squared paper to guide construction of 2D shapes - Complete tessellations of given shapes - Know the names of common 3D shapes - Use mathematical language to describe 3D shapes - Construct 3D shapes from given nets - Draw accurate nets for common 3D shapes - Find all the nets for a cube Use a net to visualise the edges (vertices) that will meet when folded	KM: Visualising 3D shapes KM: Tessellating Tess KM: Fibonacci's disappearing squares KM: Unravelling dice NRICH: Making spirals NRICH: Cut nets NRICH: Making cuboids **Learning review** www.diagnosticquestions.com

Prerequisites	Mathematical language	Agreed Common Teaching Approaches
- Know the names of common 2D shapes - Know the names of common 3D shapes - Use a protractor to measure and draw angles	Protractor Measure Nearest Construct Sketch Cube, Cuboid, Cylinder, Pyramid, Prism Net Edge, Face, Vertex (Vertices) Visualise **Notation** Dash notation to represent equal lengths in shapes and geometric diagrams Right angle notation	A prism must have a polygonal cross-section, and therefore a cylinder is not a prism. Similarly, a cone is not a pyramid. A cube is a special case of a cuboid, and a cuboid is a special case of a prism. Many pupils struggle to sketch 3D shapes. A good strategy for any type of prism is to draw the cross-section (using squares for guidance), and then draw a second identical shape offset from the first. The matching corners can then be joined with straight lines. Some dotted lines (or rubbing out of lines) will be required. NCETM: Glossary **Common approaches** *Every classroom has a set of 3D shape posters on the wall* *Models of 3D shapes to be used by all students during this unit of work* *All pupils to use 'Polydron' to explore nets of 3D shapes during this unit of work*

Reasoning opportunities and probing questions	Cross Curicular Links	Possible misconceptions
- Show me an example of a net of a cube. And another. And another … - What is wrong with this attempt at a net of a cuboid? How can it be changed? - How many different ways are there to complete these nets?		- Some pupils will read the wrong way round the scale on a typical semi-circular protractor, therefore using 180° - required angle - Some pupils may measure from the end of a ruler, rather than the start of the measuring scale - Some pupils may think that several repeats of a shape in any pattern constitutes a tessellation - When given a net of a 3D shape some pupils may think that the number of vertices of the 3D shape is found by counting the number of 'corners' on the net

Investigating properties of shapes	Link To Stage 7*	_9 hours_

KNOWLEDGE   **The Big Picture**: Properties of Shape progression map

- **compare and classify geometric shapes based on their properties and sizes and find unknown angles in any triangles, quadrilaterals, and regular polygons (ALSO 7*)**
- **draw 2-D shapes using given dimensions and angles (NEW)**
- **draw diagrams from written descriptions (NEW, ALSO 7*)**
- **illustrate and name parts of circles, including radius, diameter and circumference and know that the diameter is twice the radius**
- **identify properties of the faces, surfaces, edges and vertices of: cubes, cuboids, prisms, cylinders, pyramids, cones and spheres**

SKILLS	Suggested resources
- Know the definitions of special triangles - Know the definitions of special quadrilaterals - Classify 2D shapes using given categories; e.g. number of sides, symmetry - Know the angle sum of a triangle - Know the angle sum of a quadrilateral - Know how to find the angle sum of any polygon - Use the angle sum of a triangle to find missing angles - Find the missing angle in an isosceles triangle when only one angle is known - Use the angle sum of a quadrilateral to find missing angles - Know how to find the size of one angle in any regular polygon	KM: Shape work: Many of the activities are suitable for this unit. KM: Dotty activities KM: Investigating polygons. Tasks one and two. KM: Special polygons NRICH: Where Are They? NRICH: Round a Hexagon NRICH: Quadrilaterals KM: 6 point circles, 8 point circles and 12 point circles can be used to support and extend the above idea **Learning review** www.diagnosticquestions.com

Prerequisites	Mathematical language	Agreed Common Teaching Approaches
- Know the properties of rectangles - Know the difference between a regular and an irregular polygon - Add and subtract numbers up to three digits	Quadrilateral, Square, Rectangle, Parallelogram, (Isosceles) Trapezium, Kite, Rhombus, Delta, Arrowhead Triangle, Scalene, Right-angled, Isosceles, Equilateral Polygon, Regular, Irregular Pentagon, Hexagon, Octagon, Decagon, Dodecagon Circle, Radius, Diameter, Circumference, Centre Parallel Diagonal Angle **Notation** Dash notation to represent equal lengths in shapes and geometric diagrams Right angle notation	Ensure that pupils do not use the word 'diamond' to describe a kite, or a square that is 45° to the horizontal. 'Diamond' is not the mathematical name of any shape. A square is a special case of a rectangle. An oblong is a rectangle that is not a square. A rhombus is a special case of a parallelogram. All polygons up to 20 sides have names, although many have alternatives based on either Latin or Greek. Splitting any polygon into triangles (by drawing all diagonals from one vertex) will allow pupils to find the angle sum of the polygon. NCETM: Glossary **Common approaches** _All teachers refer to a 'delta' instead of an 'arrowhead'_ _Every classroom has a set of triangle posters and quadrilateral posters on the wall_

Reasoning opportunities and probing questions	Cross Curicular Links	Possible misconceptions
- Convince me that a rhombus is a parallelogram - Jenny writes that 'Diameter = 2 × Radius'. Kenny writes that 'Radius = 2 × Diameter'. Who is correct? - What is the same and what is different: a square and a rectangle?		- Some pupils may think that a 'regular' polygon is a 'normal' polygon - Some pupils may think that all polygons have to be regular - Some pupils may think that a square is only square if 'horizontal', and even that a 'non-horizontal' square is called a diamond - The equal angles of an isosceles triangle are not always the 'base angles' as some pupils may think

Algebraic proficiency: tinkering	Link To Stage 7*	*9 hours*

KNOWLDGE   **The Big Picture**: Algebra progression map

- **understand and use the concepts and vocabulary of expressions, equations, formulae, terms and factors (ALSO 7*)**
- **use and interpret algebraic notation, including: ab in place of a × b, 3y in place of y + y + y and 3 × y, a² in place of a × a, a³ in place of a × a × a, a/b in place of a ÷ b, brackets (ALSO 7*)**
- **simplify and manipulate algebraic expressions by collecting like terms and multiplying a single term over a bracket (ALSO 7*)**
- **substitute numerical values into formulae and expressions**
- **use conventional notation for priority of operations, including brackets**

SKILLS	Suggested resources
- Know the meaning of expression, term, formula, equation, function - Know basic algebraic notation (the rules of algebra) - Use letters to represent variables - Identify like terms in an expression - Simplify an expression by collecting like terms - Know how to multiply a (positive) single term over a bracket (the distributive law) - Substitute positive numbers into expressions and formulae - Given a function, establish outputs from given inputs - Given a function, establish inputs from given outputs - Use a mapping diagram (function machine) to represent a function - Use an expression to represent a function - Use the order of operations correctly in algebraic situations	KM: Pairs in squares. Prove the results algebraically. KM: Algebra ordering cards KM: Spiders and snakes. See the 'clouding the picture' approach KM: Use number patterns to develop the multiplying out of brackets KM: Maths to Infinity: Brackets NRICH: Your number is … NRICH: Crossed ends NRICH: Number pyramids and More number pyramids **Learning review** www.diagnosticquestions.com

Prerequisites	Mathematical language	Agreed Common Teaching Approaches
- Use symbols (including letters) to represent missing numbers - Substitute numbers into worded formulae - Substitute numbers into simple algebraic formulae - Know the order of operations **Bring on the Maths⁺: Moving on up!** Algebra: v1	Algebra Expression, Term, Formula (formulae), Equation, Function, Variable Mapping diagram, Input, Output Represent Substitute Evaluate Like terms Simplify / Collect **Notation** See key concepts above	Pupils will have experienced some algebraic ideas previously. Ensure that there is clarity about the distinction between representing a variable and representing an unknown. Note that each of the statements 4x, 42 and 4½ involves a different operation after the 4, and this can cause problems for some pupils when working with algebra. NCETM: Algebra NCETM: Glossary **Common approaches** *All pupils are expected to learn about the connection between mapping diagrams and formulae (to represent functions) in preparation for future representations of functions graphically.*

Reasoning opportunities and probing questions	Cross Curicular Links	Possible misconceptions
- Show me an example of an expression / formula / equation - Always / Sometimes / Never: 4(g+2) = 4g+8, 3(d+1) = 3d+1, a² = 2a, ab = ba - What is wrong? - Jenny writes 2a + 3b + 5a – b = 7a + 3. Kenny writes 2a + 3b + 5a – b = 9ab. What would you write? Why?		- Some pupils may think that it is always true that a=1, b=2, c=3, etc. - A common misconception is to believe that $a^2 = a \times 2 = a2$ or $2a$ (which it can do on rare occasions but is not the case in general) - When working with an expression such as 5a, some pupils may think that if a=2, then 5a = 52. - Some pupils may think that 3(g+4) = 3g+4 - The convention of not writing a coefficient of 1 (i.e. '1x' is written as 'x' may cause some confusion. In particular some pupils may think that 5h – h = 5

Exploring fractions, decimals and percentages	Link To Stage 7*		9 hours

KNOWLDGE

The Big Picture: Fractions, decimals and percentages progression map

- use common factors to simplify fractions; use common multiples to express fractions in the same denomination
- compare and order fractions, including fractions > 1
- associate a fraction with division and calculate decimal fraction equivalents [for example, 0.375] for a simple fraction [for example, 3/8] (ALSO 7*)
- recall and use equivalences between simple fractions, decimals and percentages, including in different contexts (ALSO 7*)

SKILLS	Suggested resources
• Understand that two fractions can be equivalent • Identify a common factor of two numbers • Simplify a fraction • Write a fraction in its lowest terms • Confirm that a fraction is written in its lowest terms • Compare two fractions by considering diagrams • Compare two fractions by considering equivalent fractions • Compare two top-heavy fractions • Understand that a fraction is also a way of representing a division • Know standard fraction / decimal equivalences (e.g. ½ = 0.5, ¼ = 0.25, 1/10 = 0.1) • Work out the decimal equivalents of fifths, eighths and tenths (without using a calculator) • Work out the decimal equivalents of fifths, eighths and tenths and other fractions (using a calculator) • Know standard fraction / decimal / percentage equivalences (e.g. 10%, 25%, 50%, 75%) • Work out the percentage equivalents of fifths, eighths and tenths Use the equivalence between fractions, decimals and percentages when solving problems	KM: Fraction action KM: FDP conversion KM: Carpets KM: Fraction and decimal tables NRICH: Matching fractions NRICH: Fractions made faster **Learning review** • www.diagnosticquestions.com

Prerequisites	Mathematical language	Agreed Common Teaching Approaches
• Understand the concept of a fraction as a proportion • Understand the concept of equivalent fractions • Understand the concept of fractions, decimals and percentages being equivalent • Know that a percentage means 'out of 100'	Fraction Improper fraction, Proper fraction, Vulgar fraction, Top-heavy fraction Percentage Decimal Proportion Simplify Equivalent Lowest terms **Notation** Diagonal fraction bar / horizontal fraction bar	Use language carefully to avoid later confusion: when simplifying fractions, the language 'divide by 4' should not be used in place of 'divide the top and bottom by 4'. A fraction can be divided by 4, but that is not the same as cancelling a common factor of the numerator and denominator by dividing them by 4. NRICH: Teaching fractions with understanding NCETM: Teaching fractions NCETM: Glossary **Common approaches** *All pupils are made aware that 'per cent' is derived from Latin and means 'out of one hundred'* *Teachers use the horizontal fraction bar notation at all times*

Reasoning opportunities and probing questions	Cross Curicular Links	Possible misconceptions
• Show me another fraction that is equivalent to this one. And another. And another … • Convince me that 3/8 = 0.375 • If you know that 1/10 = 0.1 = 10%, what else can you work out? • Jenny is simplifying fractions. She has the fraction 16/64. Jenny says, 'if I cancel out the sixes then 16/64 = 1/4.'. Do you agree with Jenny? Why?		• A fraction can be visualised as divisions of a shape (especially a circle) but some pupils may not recognise that these divisions must be equal in size, or that they can be divisions of any shape. • Pupils may not make the connection that a percentage is a different way of describing a proportion • Some pupils may think that simplifying a fraction just requires searching for, and removing, a factor of 2 (repeatedly)

Proportional reasoning	Link To Stage 7*	*6 hours*

KNOWLDGE **The Big Picture**: Ratio and Proportion progression map

- solve problems involving the relative sizes of two quantities where missing values can be found by using integer multiplication and division facts
- solve problems involving similar shapes where the scale factor is known or can be found
- solve problems involving unequal sharing and grouping using knowledge of fractions and multiples
- use ratio notation, including reduction to simplest form (ALSO 7*)
- divide a given quantity into two parts in a given part:part or part:whole ratio (ALSO 7*)

Return to overview

SKILLS	Suggested resources
- Identify when a comparison problem can be solved using multiplication - Identify when a comparison problem can be solved using division - Identify when a comparison problem requires both division and multiplication - Find the value of a single item in a comparison problem - Use the value of a single item to solve a comparison problem - Use knowledge of fractions to solve a sharing (or grouping) problem - Use knowledge of multiples to solve a sharing (or grouping) problem - Understand the meaning of enlargement - Understand the meaning of scale factor - Recognise when one shape is an enlargement of another - Use a scale factor to complete an enlargement - Find the scale factor for a given enlargement	NRICH: Orange Drink NRICH: Pumpkin Pie Problem NRICH: Jumping NCETM: Activity Set A NCETM: Activity Set B NCETM: Activity Set C NCETM: Activity Set D **Learning review** www.diagnosticquestions.com

Prerequisites	Mathematical language	Agreed Common Teaching Approaches
- Recall multiplication facts for multiplication tables up to 12 × 12 - Recall division facts for multiplication tables up to 12 × 12 - Find fractions of an amount - Find multiples of a given number	Proportion Quantity Integer Similar (shapes) Enlargement Scale factor Group Share Multiples	Any work on enlargement should only include enlargements using a scale factor. The concept of a centre of enlargement is a future development. NCETM: The Bar Model NCETM: Multiplicative reasoning NCETM: Glossary **Common approaches** *All pupils are explicitly taught to use the bar model as a way to represent a problem involving proportional reasoning*

Reasoning opportunities and probing questions	Cross Curicular Links	Possible misconceptions
(Given a recipe for 4 people) show me an amount of food that is needed for 8 people, 6 people, 9 people. Show me an amount of food that is needed for a number of people of your choice. And another. And another … - Convince me that the second shape is an enlargement of the first shape - Kenny has no sweets. Jenny gives 1/3 of her sweets to Kenny. Kenny now has 18 sweets. Kenny thinks that Jenny had 54 sweets to start with. Kenny is wrong. Explain why. NCETM: Ratio and Proportion Reasoning		- Many pupils will want to identify an additive relationship between two quantities that are in proportion and apply this to other quantities in order to find missing amounts - When finding a fraction of an amount some pupils may try to use a rule formed without the necessary understanding. As a result they will muddle the operations, dividing by the numerator and multiplying by the denominator. - When constructing an enlargement some pupils may only apply the scale factor in one dimension; for example, 'enlarging' a 2 by 4 rectangle by a scale factor of 2 and drawing a 2 by 8 rectangle.

www.kangaroomaths.com

Stage 6: Page 10

Pattern sniffing	Link To Stage 7*	_3 hours_

KNOWLDGE

- **generate and describe linear number sequences**
- **generate terms of a sequence from a term-to-term rule**

The Big Picture: Algebra progression map

SKILLS	Suggested resources
- Use the vocabulary of sequences - Recognise a linear sequence - Describe a number sequence - Find the next term in a linear sequence - Find a missing term in a linear sequence Generate a linear sequence from its description (Introduce the nth term)	KM: Maths to Infinity: Sequences NRICH: Times Tables Shifts NRICH: Domino Sets NCETM: Activity B: Sticky Triangles NCETM: Activity D: Generating Sequences **Learning review** www.diagnosticquestions.com

Prerequisites	Mathematical language	Agreed Common Teaching Approaches
- Count forwards and backwards in tens (hundreds, thousands) from any positive number up to 10 000 (100 000, 1 000 000) - Count forwards and backwards through zero	Pattern Sequence Linear Term Ascending Descending	Pupils have counted forwards and backwards in previous years and units, but this is the first time that the concept of sequences appears specifically. The language 'term-to-term rule' should not be introduced until Stage 7. NCETM: Algebra NCETM: Glossary **Common approaches** _Teachers and pupils refer to numbers less than zero as 'negative' numbers and not 'minus' numbers_

Reasoning opportunities and probing questions	Cross Curicular Links	Possible misconceptions
- Show me a (ascending/descending) linear sequence. And another. And another. - Kenny thinks that 2, 4, 8, 16, … is a linear example. Do you agree? Explain your answer. - Create a linear sequence with a 3rd term of '8'. - Show me a linear sequence where the rule to get from one term to the next is 'add 3'. And another. And another. NCETM: Algebra Reasoning		- Some pupils may think linear sequences are only ascending. - Some pupils may think that any sequence that can be described by a rule to get from one term to the next is a linear sequence, e.g. 2, 4, 8, 16, … - Some pupils may not appreciate that both a starting number and a rule to find the next term are required in order to describe a sequence in full.

Measuring space	Link To Stage 7*	3 hours

KNOWLDGE **The Big Picture**: Measurement and mensuration progression map

- use, read, write and convert between standard units, converting measurements of length, mass, volume and time from a smaller unit of measure to a larger unit, and vice versa, using decimal notation to up to three decimal places (ALSO 7*)
- convert between miles and kilometres (NEW, ALSO 7*)
- use standard units of mass, length, time, money and other measures using decimal quantities where appropriate (ALSO 7*)

Return to overview

SKILLS	Suggested resources
• Convert between non-adjacent metric units; e.g. kilometres and centimetres • Use decimal notation up to three decimal places when converting metric units • Convert between Imperial units; e.g. feet and inches, pounds and ounces, pints and gallons • Solve problems involving converting between measures State conclusions using the correct notation and units	KM: Weighing up the options NRICH: Place Your Orders NRICH: Thousands and Millions NCETM: Activity E : A little bit of history - Marco Polo **Learning review** • www.diagnosticquestions.com

Prerequisites	Mathematical language	Agreed Common Teaching Approaches
• Convert between adjacent metric units of length, mass and capacity • Know rough equivalents between inches and cm, feet and cm, kg and lb, pint and ml • Use decimal notation to two decimal places when converting between metric unit	Length, distance Mass, weight Volume Capacity Metre, centimetre, millimetre Tonne, kilogram, gram, milligram Litre, millilitre Hour, minute, second Inch, foot, yard Pound, ounce Pint, gallon **Notation** Abbreviations of units in the metric system: m, cm, mm, kg, g, l, ml Abbreviations of units in the Imperial system: lb, oz	Weight and mass are distinct though they are often confused in everyday language. Weight is the force due to gravity, and is calculated as mass multiplied by the acceleration due to gravity. Therefore weight varies due to location while mass is a constant measurement. The prefix 'centi-' means one hundredth, and the prefix 'milli-' means one thousandth. These words are of Latin origin. The prefix 'kilo-' means one thousand. This is Greek in origin. Conversion of volumes will be covered in the 'calculating space' unit. NCETM: Glossary **Common approaches** *Every classroom has a sack of sand (25 kg), a bag of sugar (1 kg), a cheque book (1 cheque is 1 gram), a bottle of water (1 litre, and also 1 kg of water) and a teaspoon (5 ml)*

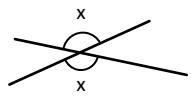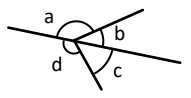| Reasoning opportunities and probing questions	Cross Curicular Links	Possible misconceptions
• Show me a metric (imperial) unit of measure. And another. And another. • Kenny thinks that 2.5km = 25 000 cm. Do you agree with Kenny? Explain your answer. • Convince me that 4.25kg does not equal 425g. NCETM: Measurement Reasoning		• Some pupils may apply an incorrect understanding that there are 100 minutes in a hour when solving problems • Some pupils may struggle when converting between 12- and 24-hour clock notation; e.g. thinking that 15:00 is 5 o' clock • Some pupils may apply incorrect beliefs about place value, such as 2.3 × 10 = 2.30. • Many conversions within the metric system rely on multiplying and dividing by 1000. The use of centimetres as an 'extra unit' within the system breaks this pattern. Consequently there is a frequent need to multiply and divide by 10 or 100, and this can cause confusion about the connections that need to be applied.

Investigating angles	Link To Stage 7*	*3 hours*

KNOWLEDGE

- **recognise angles where they meet at a point, are on a straight line, or are vertically opposite, and find missing angles**
- **apply the properties of angles at a point, angles at a point on a straight line, vertically opposite angles (ALSO 7*)**

SKILLS	Suggested resources
- Identify angles that meet at a point - Identify angles that meet at a point on a line - Identify vertically opposite angles - Know that vertically opposite angles are equal - Use known facts to find missing angles Explain reasoning	KM: Maths to Infinity: Lines and Angles **Learning review** - www.diagnosticquestions.com

Prerequisites	Mathematical language	Agreed Common Teaching Approaches
- Know that angles are measured in degrees - Know that angles in a full turn total 360°, and angle in half a turn must total 180° - Estimate the size of angles	Angle Degrees Right angle Acute angle Obtuse angle Reflex angle Protractor Vertically opposite **Notation** Right angle notation Arc notation for all other angles The degree symbol (°)	The exact reason for there being 360 degrees in a full turn is unknown. There are various theories including it being an approximation of the 365 days in a year and resultant apparent movement of the sun, and the fact that it has so many factors. The SI unit for measuring angles in the radian (2π radians in a full turn). Napoleon experimented with the decimal degree, or grad (400 grads in a full turn) NCETM: Glossary **Common approaches** *All pupils know how to to use a 180° and a 360° protractor.*

Reasoning opportunities and probing questions	Cross Curicular Links	Possible misconceptions
- Show a pair of possible values for a and b. And another. And another. - Convince me that the sum of angles on a straight line is 180°. - Show a possible set of values for a, b, c and d. And another. And another. - Convince me that the sum of angles around a point is 360°. - Convince me that (vertically) opposite angles are equal. - Kenny thinks that the sum of opposite angles is 180°. Do you agree? Explain your answer. NCETM: Geometry - Properties of Shapes Reasoning		- Some pupils may think that these angles are not equal as they are not 'vertical'. - Some pupils may think that angles that are 'roughly' opposite are always equal, e.g. a = c

Calculating fractions, decimals and percentages	Link To Stage 7*	*9 hours*

KNOWLDGE

The Big Picture: Fractions, decimals and percentages progression map

- add and subtract fractions with different denominators and mixed numbers, using the concept of equivalent fractions (ALSO 7*)
- multiply simple pairs of proper fractions, writing the answer in its simplest form [for example, $^1/_4 \times {}^1/_2 = {}^1/_8$] (ALSO 7*)
- divide proper fractions by whole numbers [for example, $^1/_3 \div 2 = {}^1/_6$] (also 7*)
- multiply one-digit numbers with up to two decimal places by whole numbers (ALSO 7*)
- solve problems involving the calculation of percentages [for example, of measures, and such as 15% of 360] and the use of percentages for comparison

Return to overview

SKILLS

- Add (subtract) fractions with different denominators
- Add (subtract) a mixed number and a fraction, including with different denominators
- Add (subtract) mixed numbers, including with different denominators
- Multiply a proper fraction by a proper fraction
- Divide a proper fraction by a whole number
- Simplify the answer to a calculation when appropriate
- Multiply U.t by U
- Multiply U.th by U
- Find 10% of a quantity
- Use non-calculator methods to find a percentage of an amount
- Use decimal or fraction equivalents to find a percentage of an amount where appropriate
  Solve problems involving the use of percentages to make comparisons

Suggested resources

NRICH: Fractions Jigsaw
NRICH: Peaches Today, Peaches Tomorrow
NRICH: Andy's Marbles
NRICH: Would you Rather?

Learning review
- www.diagnosticquestions.com

Prerequisites

- Convert between mixed numbers and improper fractions
- Find equivalent fractions
- Add and subtract fractions when one denominator is a multiple of the other
- Multiply a proper fraction by a whole number
- Use the formal written method of short multiplication
- Know the effect of multiplying and dividing by 10 and 100
- Know percentage equivalents of $^1/_2$, $^1/_4$, $^3/_4$, $^1/_5$, $^2/_5$, $^4/_5$

Bring on the Maths⁺: Moving on up!
Fractions, decimals & percentages: #5

Mathematical language

Mixed number
Equivalent fraction
Simplify, cancel
Lowest terms
Proper fraction, improper fraction, top-heavy fraction, vulgar fraction
Numerator, denominator
Percent, percentage

Notation
Mixed number notation
Horizontal / diagonal bar for fractions

Agreed Common Teaching Approaches

Use of a fraction wall to visualise multiplying fractions and dividing fractions by a whole number. For example, pupils need to read calculations such as $\frac{1}{4} \times \frac{1}{2}$ as $\frac{1}{4}$ multiplied by $\frac{1}{2}$ and therefore, $\frac{1}{2}$ of $\frac{1}{4} = \frac{1}{8}$; $\frac{4}{10} \div 2$ as $\frac{4}{10}$ divided by 2 and therefore $\frac{2}{10}$.
NCETM: The Bar Model
NCETM: Teaching fractions
NCETM: Fractions videos
NCETM: Glossary

Common approaches
When multiplying a decimal by a whole number pupils are taught to use the corresponding whole number calculation as a general strategy.
When adding and subtracting mixed numbers pupils are taught to convert to improper fractions as a general strategy.
Pupils are encouraged to find and use 10% of an amount.

Reasoning opportunities and probing questions

- Show me an 'easy' ('difficult') pair of fractions to add (subtract). And another. And another.
- Kenny thinks that $\frac{7}{10} - \frac{2}{7} = \frac{5}{3} = 1\frac{2}{3}$. Do you agree with Kenny?
- Jenny thinks that you can only multiply fractions if they have the same common denominator. Do you agree with Jenny? Explain.
- Benny thinks that $\frac{4}{10} \div 2 = \frac{4}{5}$. Do you agree with Kenny? Explain.
- Lenny says *'20% of £60 is £3 because 60 ÷ 20 = 3'*. Do you agree?
NCETM: Fractions Reasoning
NCETM: Ratio and Proportion Reasoning

Cross Curicular Links

Possible misconceptions

- Some pupils may think that you simply can simply add/subtract the whole number part of mixed numbers and add/subtract the fractional art of mixed numbers when adding/subtracting mixed numbers, e.g. $3\frac{1}{3} - 2\frac{1}{2} = 1\frac{-1}{6}$
- Some pupils may make multiplying fractions over complicated by applying the same process for adding and subtracting of finding common denominators.
- Some pupils may think that as you divide by 10 to find 10%, you divided by 15 to find 15%, divide by 20 to find 20%, etc.

www.kangaroomaths.com

Stage 6: Page 14

Solving equations and inequalities	Link To Stage 7*	*3 hours*

KNOWLEDGE   The Big Picture: Algebra progression map

- **enumerate possibilities of combinations of two variables**
- **express missing number problems algebraically**
- **find pairs of numbers that satisfy an equation with two unknowns**

SKILLS	Suggested resources
- Solve missing number problems expressed in words - Find a solution to a missing number problem with two unknowns - Find all combinations of two variables that solve a missing number problem with two unknowns - Know the basic rules of algebraic notation - Express missing number problems algebraically Solve missing number problems expressed algebraically	NRICH: Plenty of Pens NRICH: Your Number Is… NRICH: Number Pyramids NCETM: Activity A: Racetrack and Design a board game NCETM: Activity E: Matchbox Algebra **Learning review** - www.diagnosticquestions.com

Prerequisites	Mathematical language	Agreed Common Teaching Approaches
- Use symbols to represent variables in a formula	Algebra, algebraic, algebraically Symbol Expression Variable Substitute Equation Unknown Enumerate **Notation** The lower case and upper case of a letter should not be used interchangeably when worked with algebra Juxtaposition is used in place of '×'. $2a$ is used rather than $a2$. Division is written as a fraction	The word 'algebra' comes from the title of a book by the Persian mathematician, al-Khwārizmī, who lived in modern-day Baghdad about 1200 years ago. Al-kitāb al-mukhtaṣar fī ḥisāb **al-ğabr** wa'l-muqābala was a book that promoted the idea of solving equations by a method of balancing. Avoid fruit salad algebra (see possible misconceptions). NCETM: The Bar Model NCETM: Algebra NCETM: Glossary **Common approaches** *Use 'think of a number' problems to introduce the idea of 'n' standing for any number (a variable). For example:* - *Think of a number, double it, add 8, halve it and finally subtract your starting number.* *The answer is always 4. n → 2n → 2n + 8 → n + 4 → 4.* *Encourage students to try again with different numbers, including large, small, negative, fractional or decimal. Also try varying the instructions to end up with a different number every time.*

Reasoning opportunities and probing questions	Cross Curicular Links	Possible misconceptions
- $a + b = 15$. Show me a pair of values for a and b. And another. And another. - $p + q = 7$. Show me a pair of values for p and q that no one else will think of. And another. And another. - Kenny thinks that 'b^2' is the same as '$2b$' because when $b = 2$, $b^2 = 4$ and $2b = 4$. Do you agree with Kenny? Explain your answer. - Jenny thinks that $7 + 2a = 9a$. Do you agree with Jenny? Explain your answer. NCETM: Algebra Reasoning		- Some pupils may think that variables have a set value, such as $a = 1$, $b = 2$, $c = 3$, $d = 4$, etc. (especially if they have done lots of poorly designed treasure hunts/codes) – this will lead to problems such as thinking 'b^2' is the same as '$2b$' because when $b = 2$, $b^2 = 4$ and $2b = 4$. - Using the idea of 'apples' and 'bananas' to explain $a + b = 14$ can lead to misconceptions about the use of letters as variables. - Some students may think that the variables have to be positive integers (whole numbers)

Calculating space	Link To Stage 7*	*9 hours*

KNOWLEDGE

The Big Picture: Measurement and mensuration progression map

- **recognise that shapes with the same areas can have different perimeters and vice versa**
- **calculate the area of parallelograms and triangles**
- **calculate, estimate and compare volume of cubes and cuboids using standard units, including cubic centimetres (cm³) and cubic metres (m³), and extending to other units [for example, mm³ and km³]**
- **recognise when it is possible to use formulae for area and volume of shape**
- **solve problems involving the calculation and conversion of units of measure, using decimal notation up to three decimal places where appropriate**
- **calculate perimeters of 2D shapes (ALSO 7*)**
- **calculate surface area of cuboids**

SKILLS	Suggested resources
- Recognise that shapes with the same areas can have different perimeters and vice versa - Know that the area of a parallelogram is given by the formula area = base × height - Know that the area of a triangle is given by the formula area = ½ × base × height = base × height ÷ 2 = $\frac{bh}{2}$ - Know that the volume of a cuboid is given by the formula volume = length × width × height - Calculate the area of a parallelogram (triangle) - Recognise when it is possible to use a formula for the area of a shape - Estimate the volume of cubes and cuboids - Choose appropriate units of volume - Calculate the volume of a cuboid - Recognise when it is possible to use a formula for the volume of a shape - Convert between metric units of area in simple cases Convert between metric units of volume in simple cases	KM: Fibonacci's disappearing squares KM: Dissections deductions NCETM: Activity C: Through the window **Learning review** www.diagnosticquestions.com

Prerequisites	Mathematical language	Agreed Common Teaching Approaches
- Know the meaning of perimeter (area, volume, capacity) - Know that the area of a rectangle is given by the formula area = length × width - Know that area can be measured using square centimetres or square metres, and the abbreviations cm² and m² - Know that volume is measured in cubes **Bring on the Maths⁺: Moving on up!** Measures: #4, #5	Perimeter, area, volume, capacity Square, rectangle, parallelogram, triangle Composite rectilinear Polygon Cube, cuboid Millimetre, Centimetre, Metre, Kilometre Square millimetre, square centimetre, square metre, square kilometre Cubic centimetre, centimetre cube Formula, formulae Convert Length, breadth, depth, height, width **Notation** Abbreviations of units in the metric system: km, m, cm, mm, mm², cm², m², km², mm³, cm³, km³	In this unit, 'volumes of shapes' refers only to cubes and cuboids. Ensure that pupils make connections with the area of a rectangle work in Stage 5, in particular the importance of the perpendicular height. Note that there are several different ways of stating the area of a triangle and this can cause confusion NCETM: Glossary **Common approaches** *Pupils derive practically the formulae for area of parallelogram and triangle by dissecting rectangles* *Pupils derive the formula for the area of a parallelogram first. They then use this to help derive the formula for the area of an obtuse-angled triangle.* *Every classroom has a set of area posters on the wall* *Pupils use the area of a triangle as given by the formula area = $\frac{bh}{2}$*

Reasoning opportunities and probing questions	Cross Curicular Links	Possible misconceptions
'Show me' an example of when you would measure volume using km³ - Convince me that the area of a parallelogram is found using base × height (Given a triangle with base labelled 8 cm, height 5 cm, slope height 6 cm) Kenny thinks that the area is 40 cm², Lenny thinks it is 20 cm², Jenny thinks it is 240 cm² and Benny thinks it is 24 cm². Who do you agree with? Explain why.		- Some pupils may use the sloping height when finding the areas of parallelograms and triangles - Some pupils may think that the area of a triangle is found using area = base × height - Some pupils may think that you multiply all the numbers to find the area of a shape

Checking, approximating and estimating	Link To Stage 7*	*3 hours*

KNOWLEDGE

The Big Picture: Number and Place Value progression map

- **solve problems which require answers to be rounded to specified degrees of accuracy**
- **use estimation to check answers to calculations and determine, in the context of a problem, an appropriate degree of accuracy**
- **round any whole number to a required degree of accuracy**

SKILLS	Suggested resources
• Approximate any number by rounding to the nearest 1 000 000 • Approximate any number by rounding to a specified degree of accuracy; e.g. nearest 20, 50 • Understand estimating as the process of finding a rough value of an answer or calculation • Use estimation to predict the order of magnitude of the solution to a (decimal) calculation • Check the order of magnitude of the solution to a (decimal) calculation • Estimate multiplication calculations that involve multiplying up to four-digit numbers by a two-digit number • Estimate division calculations that involve dividing up to a four-digit number by a two-digit number Estimate multiplication calculations that involve multiplying numbers with up to two decimal places by whole numbers	KM: Stick on the Maths CALC6: Checking results KM: Maths to Infinity Rounding NRICH: Four Go NCETM: Activity A(i) NCETM: Activity G **Learning review** • www.diagnosticquestions.com

Prerequisites	Mathematical language	Agreed Common Teaching Approaches
• Approximate any number by rounding to the nearest 10, 100 or 1000, 10 000 or 100 000 • Approximate any number with one or two decimal places by rounding to the nearest whole number • Approximate any number with two decimal places by rounding to the one decimal place • Estimate addition (subtraction) calculations with up to four digits	Approximate (noun and verb) Round Decimal place Check Solution Answer Estimate (noun and verb) Order of magnitude Accurate Accuracy **Notation** The approximately equal symbol (≈)	This unit is an opportunity to develop and practice calculation skills with a particular emphasis on checking, approximating or estimating the answer. Pupils should use numbers up to 10 000 000 in this unit. Pupils should be able to round to other specified degrees of accuracy, but not to a specified number of significant figures, which is introduced in Stage 7. Also see big pictures: Calculation progression map and Fractions, decimals and percentages progression map NCETM: Glossary **Common approaches** *All pupils are taught to visualise rounding through the use a number line*

Reasoning opportunities and probing questions	Cross Curicular Links	Possible misconceptions
• Convince me that 67 rounds to 60 to the nearest 20 • Convince me that 1 579 234 rounds to 2 million to the nearest million • Jenny writes 1359 ÷ 18 ≈ 7.55. Comment on Jenny's approximation. • Lenny writes 2.74 × 13 ≈ 26. Do you agree with Lenny? Explain your answer. NCETM: Place Value Reasoning		• Some pupils may truncate instead of round • When checking the order of magnitude of a division calculation some pupils may apply incorrect reasoning about the effect of increasing the divisor by a factor of 10, thinking that it also makes the solution greater by a factor of 10; e.g. 1400 ÷ 20: 1400 ÷ 2 = 700 so 1400 ÷ 20 = 7000. • Some pupils may round down at the half way point, rather than round up.

Mathematical movement	Link To Stage 7*	*3 hours*

KNOWLEDGE

- **describe positions on the full coordinate grid (all four quadrants)**
- **work with co-ordinates in all four quadrants (NEW)**
- **draw and translate simple shapes on the coordinate plane, and reflect them in the axes**
- **solve geometric problems on coordinate axes**
- **describe translations as 2D vectors (ALSO 7*)**

SKILLS	Suggested resources
• Use coordinates to describe the position of a point in all four quadrants • Use coordinates to write the position of a point in all four quadrants • Construct a 2-D coordinate grid (all four quadrants) • Use coordinates to plot the position of a point in any of the four quadrants • Use coordinates to plot a set of points to construct a polygon • Solve problems involving coordinates • Carry out a translation Carry out a reflection using one of the axes as a mirror line	KM: Stick on the Maths ALG2: Coordinates in four quadrants NRICH: Cops and Robbers NRICH: Eight Hidden Squares NRICH: Coordinate Tan NRICH: Transformation Tease NCETM: Activity B - Battleships **Learning review** • www.diagnosticquestions.com

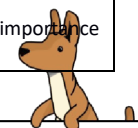| Prerequisites	Mathematical language	Agreed Common Teaching Approaches
• Use coordinates in the first quadrant • Identify a translation • Carry out a translation in the first quadrant • Identify a reflection • Carry out a reflection in the first quadrant using mirror lines parallel to the axes • Know the meaning of 'congruent', 'congruence', 'object', 'image' **Bring on the Maths+: Moving on up!** Properties of shapes: #2 Position and direction: #1	2-D Grid Axis, axes, x-axis, y-axis Origin Quadrant (Cartesian) coordinates Point Translation Reflection Transformation Object, Image Congruent, congruence **Notation** Cartesian coordinates should be separated by a comma and enclosed in brackets (x, y)	The main focus of this unit is to develop understanding of coordinates in all four quadrants Note that pupils are not yet expected to use an algebraic description of a mirror line (such as x = 3). The French mathematician Rene Descartes introduced Cartesian coordinates in the 17th century. It is said that he thought of the idea while watching a fly moving around on his bedroom ceiling. Other coordinate systems include grid references, polar coordinates and spherical coordinates. There are other types of mathematical movement that pupils will learn about in future stages. The group name for these movements is 'transformations'. NCETM: Glossary **Common approaches** *Teachers do not use the phrase 'along the corridor and up the stairs' as it can encourage a mentality of only working in the first quadrant. Later, pupils will have to use coordinates in all four quadrants. A more helpful way to remember the order of coordinates is 'x is a cross, wise up!'* *Teachers use the language 'negative number', and not 'minus number', to avoid future confusion with calculations.*

Reasoning opportunities and probing questions	Cross Curicular Links	Possible misconceptions
• (Given a grid with the point (-3, 4) indicated) Benny describes this point as (-3, 4). Jenny describes the point as (4, -3). Who do you agree with? Why? • Two vertices of a rectangle are (-1, 2) and (4, -2). What could the other two vertices be? How many solutions can you find? • Convince me that (-2, 3) is in the second quadrant NCETM: Geometry: Position Direction and Movement Reasoning		• When describing or carrying out a translation, some pupils may count the squares between the two shapes rather than the squares that describe the movement between the two shapes. • When reflecting a triangle some students may draw a translation • When carrying out a reflection some pupils may think that the object and image should be an equal distance from the edge of the grid, rather than an equal distance form the mirror line. • Some pupils will confuse the order of x-coordinates and y-coordinates • When constructing axes, some pupils may not realise the importance of equal divisions on the axes

Presentation of data	Link To Stage 7*		3 hours

KNOWLEDGE   **The Big Picture**: Statistics progression map

- interpret and construct pie charts and line graphs and use these to solve problems

SKILLS

- Understand that pie charts are used to show proportions
- Make statements about proportions shown in a pie charts
- Make statements to compare proportions in pie charts
- Use additional information to make statements about frequencies in pie charts
- Use a table of frequencies to work out the angle for a slice in a pie chart
- Construct a pie chart by measuring angles
- Identify the scale used on the axes of a graph
- Read values from a line graph involving scaling
- Use scaling when constructing line graphs

Answer two-step questions about data in line graphs (e.g. 'How much more?')

Suggested resources

KM: Stick on the Maths HD6: Graphs and diagrams
NRICH: Match the Matches
NRICH: Graphing Number Patterns
NCETM: A little bit of history (Britain since 1945)

Learning review

- www.diagnosticquestions.com

Prerequisites

- Measure and construct angles using a protractor
- Interpret and construct a simple line graph

Mathematical language

Data
Scale
Axis, axes
Graph
Frequency
Time graph, Time series
Line graph
Pie chart
Sector
Angle
Protractor
Degrees
Maximum, minimum

Agreed Common Teaching Approaches

In Stage 6, when constructing pie charts the total of the frequencies is always a factor of 360 More complex cases are included in later stages.

William Playfair, a Scottish engineer and economist, introduced the line graph in 1786. He also introduced the pie chart in 1801.

NCETM: Glossary

Common approaches
Pie charts are constructed by calculating the angle for each section by dividing 360 by the total frequency, and not using percentages.
The angle for the first section is measured from a vertical radius. Subsequent sections are measured using the boundary line of the previous section.

Reasoning opportunities and probing questions

- Show me a pie chart representing the following information: Blue (25%), Red (over 50%), Yellow (the rest). And another. And another.
- Always / Sometimes / Never: Pie charts are constructed in a clockwise direction
- Always / Sometimes / Never: The larger the size of the pie chart, the greater the total frequency
- Kenny says 'If two pie charts have the same section then the amount of data the section represents is the same in each pie chart.' Do you agree with Kenny? Explain your answer.

NCETM: Statistics Reasoning

Cross Curicular Links

Possible misconceptions

- Some pupils may think that a line graph is appropriate for discrete data
- Some pupils may think that each square on the grid used represents one unit
- Some pupils may confuse the fact that the sections of the pie chart total 100% and 360°

Measuring data	Link To Stage 7*		3 hours

KNOWLEDGE

The Big Picture: Statistics progression map

- **calculate and interpret the mean as an average**
- **interpret, analyse and compare the distributions of data sets from univariate empirical distributions through appropriate measures of central tendency (median, mean and mode) and spread (range) (ALSO 7*)**

Return to overview

SKILLS	Suggested resources
• Understand the meaning of 'average' as a typicality (or location) • Understand the mean as a measure of typicality (or location) • Interpret the mean as a way of levelling the data • Calculate the mean of a set of data • Choose an appropriate approximation when required Use the mean to find a missing number in a set of data	KM: Maths to Infinity: Averages, Charts and Tables NRICH: Birdwatch NRICH: Probably … NRICH: Same or Different? NCETM: A little bit of history (Britain since 1945) **Learning review** • www.diagnosticquestions.com

Prerequisites	Mathematical language	Agreed Common Teaching Approaches
• Approximate a number by rounding to a given number of decimal places	Average Mean Measure Data Statistic Statistics Approximate Round	The word 'average' is often used synonymously with the mean, but it is only one type of average. In fact, there are several different types of mean (the one in this unit properly being named as the 'arithmetic mean'). Other types of average, including the mode and the median, are introduced in later stages. NCETM: Glossary **Common approaches** *Always use brackets when writing out the calculation for a mean, e.g. (2 + 3 + 4 + 5) ÷ 4 = 14 ÷ 4 = 3.5*

Reasoning opportunities and probing questions	Cross Curicular Links	Possible misconceptions
• Always / Sometimes / Never: The mean is a whole number. • Kenny is working out the mean of 2, 3, 4 and 5. He calculates 2 + 3 + 4 + 5 ÷ 4 = 10.25. Do you agree with Kenny? Explain your answer. • The average number of children per family (Married Couples, 2012) is 1.8. Convince me that this statement makes sense. NCETM: Statistics Reasoning		• If using a calculator some pupils may not use the '=' symbol (or brackets) correctly; e.g. working out the mean of 2, 3, 4 and 5 as 2 + 3 + 4 + 5 ÷ 4 = 10.25. • Some pupils may think the average is always the middle number • Some pupils may think that the mean must be a whole number • Some pupils may not realise that the mean must lie within the range of the data set.

Understanding risk I	Link To Stage 7*		*3 hours*

KNOWLEDGE   **The Big Picture**: Probability progression map

- **Relate relative expected frequencies to theoretical probability, using appropriate language and the 0-1 probability scale (NEW)**
- **apply the property that the probabilities of an exhaustive set of outcomes sum to one**

SKILLS	Suggested resources
- Know that probability is a way of measuring likeliness - Know and use the vocabulary of probability - Understand the use of the 0-1 scale to measure probability - Assess likeliness and place events on a probability scale - Work out theoretical probabilities for events with equally likely outcomes - Know how to represent a probability - Know that the sum of probabilities for all outcomes is 1 Apply the fact that the sum of probabilities for all outcomes is 1	KM: Probability scale and slideshow version KM: Probability loop cards NRICH: Dice and spinners interactive **Learning review** - www.diagnosticquestions.com

Prerequisites	Mathematical language	Agreed Common Teaching Approaches
- Understand the equivalence between fractions, decimals and percentages - Compare fractions, decimals or percentages - Simplify a fraction by cancelling common factors	Probability, Theoretical probability Event Outcome Impossible, Unlikely, Evens chance, Likely, Certain Equally likely Mutually exclusive Exhaustive Possibility space Experiment **Notation** Probabilities are expressed as fractions, decimals or percentage. They should not be expressed as ratios (which represent odds) or as words	It is not immediately apparent how to use words to label the middle of the probability scale. 'Evens chance' is a common way to do so, although this can be misleading as it could be argued that there is an even chance of obtaining any number when rolling a fair die. NRICH: Introducing probability NRICH: Why Do People Find Probability Unintuitive and Difficult? NCETM: Glossary **Common approaches** *Every classroom has a display of a probability scale labeled with words and numbers. Pupils create events and outcomes that are placed on this scale.*

Reasoning opportunities and probing questions	Cross Curicular Links	Possible misconceptions
- Show me an example of an event and outcome with a probability of 0. And another. And another… - Always / Sometimes / Never: if I pick a card from a pack of playing cards then the probability of picking a club is ¼ - Label this (eight-sided) spinner so that the probability of scoring a 2 is ¼.		- Some pupils will initially think that, for example, the probability of it raining tomorrow is ½ as it either will or it won't. - Some students may write a probability as odds (e.g. 1:6 or '1 to 6'). There is a difference between probability and odds, and therefore probabilities must only be written as fractions, decimals or percentages. - Some pupils may think that, for example, if they flip a fair coin three times and obtain three heads, then it must be more than likely they will obtain a head next.