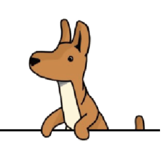
Mathematics overview: Stage 8*

Unit	Hours
Numbers and the number system	12
Calculating	12
Visualising and constructing	12
Algebraic proficiency: tinkering	12
Exploring fractions, decimals and percentages	4
Proportional reasoning	12
Pattern sniffing	4
Investigating angles	8
Calculating fractions, decimals and percentages	8
Solving equations and inequalities	4
Calculating space	12

KNOWLEDGE

- use the concepts and vocabulary of prime numbers, highest common factor, lowest common multiple, prime factorisation, including using product notation and the unique factorisation theorem
- interpret standard form A × 10n, where 1 ≤ A < 10 and n is an integer
- calculate with standard form A × 10n, where 1 ≤ A < 10 and n is an integer (ALSO 9*)
- apply the four operations, including formal written methods, to integers, decimals and simple fractions (proper and improper), and mixed numbers – all both positive and negative
- use conventional notation for priority of operations, including brackets, powers, roots and reciprocals
- calculate with roots, and with integer indices
- use inequality notation to specify simple error intervals due to truncation or rounding
- apply and interpret limits of accuracy
- measure line segments and angles in geometric figures, including interpreting maps and scale drawings and use of bearings (ALSO 9*)
- interpret plans and elevations of 3D shapes (ALSO 7*)
- use scale factors, scale diagrams and maps
- identify, describe and construct similar shapes, including on coordinate axes, by considering enlargement
- use the standard ruler and compass constructions (perpendicular bisector of a line segment, constructing a perpendicular to a given line from/at a given point, bisecting a given angle)
- use and interpret algebraic notation, including: a^2b in place of a × a × b, coefficients written as fractions rather than as decimals
- understand and use the concepts and vocabulary of factors
- rearrange formulae to change the subject
- simplify and manipulate algebraic expressions by expanding products of two binomials and factorising quadratic expressions of the form $x^2 + bx + c$
- argue mathematically to show algebraic expressions are equivalent, and use algebra to support and construct arguments (NEW)
- translate simple situations or procedures into algebraic expressions or formulae (NEW)#
- work interchangeably with terminating decimals and their corresponding fractions (such as 3.5 and 7/2 or 0.375 or 3/8) (ALSO 7*)
- express the division of a quantity into two parts as a ratio; apply ratio to real contexts and problems (such as those involving conversion, comparison, scaling, mixing, concentrations)
- understand and use proportion as equality of ratios
- express a multiplicative relationship between two quantities as a ratio or a fraction
- relate ratios to fractions and to linear functions
- use compound units (such as speed, rates of pay, unit pricing, density and pressure)
- change freely between compound units (e.g. speed, rates of pay, prices) in numerical contexts
- generate terms of a sequence from either a term-to-term or a position-to-term rule (ALSO 7*)
- deduce expressions to calculate the nth term of linear sequences (ALSO 7*)
- recognise and use Fibonacci type sequences, quadratic sequences
- derive and use the sum of angles in a triangle (e.g. to deduce and use the angle sum in any polygon, and to derive properties of regular polygons)
- use the basic congruence criteria for triangles (SSS, SAS, ASA, RHS) (ALSO 9*)
- interpret fractions and percentages as operators
- work with percentages greater than 100%
- solve problems involving percentage change, including original value problems, and simple interest including in financial mathematics (ALSO 7*)
- calculate exactly with fractions
- solve linear equations with the unknown on both sides of the equation (ALSO, 7*)



<table>
<tr><td>Algebraic proficiency: visualising</td><td>12</td><td rowspan="5">
• find approximate solutions to linear equations using a graph
• solve, in simple cases, two linear simultaneous equations in two variables algebraically (ALSO, 9*)
• derive an equation (or two simultaneous equations), solve the equation(s) and interpret the solution
• find approximate solutions to simultaneous equations using a graph
• solve linear inequalities in two variables (NEW)
• compare lengths, areas and volumes using ratio notation
• calculate perimeters of 2D shapes, including circles
• know the formulae: circumference of a circle = $2\pi r = \pi d$, area of a circle = πr^2
• calculate areas of circles and composite shapes
• know the formulae for: Pythagoras' theorem, $a^2 + b^2 = c^2$, and apply it to find lengths in right-angled triangles in two dimensional figures
• know and apply formulae to calculate volume of right prisms (including cylinders)
• identify and apply circle definitions and properties, including: tangent, arc, sector and segment
• identify and interpret gradients and intercepts of linear functions graphically and algebraically
• recognise, sketch and interpret graphs of linear functions and simple quadratic functions
• plot and interpret graphs and graphs of non-standard (piece-wise linear) functions in real contexts, to find approximate solutions to problems such as simple kinematic problems involving distance and speed
• use the form $y = mx + c$ to identify parallel lines
• interpret the gradient of a straight line graph as a rate of change
• enumerate sets and combinations of sets systematically, using tree diagrams (ALSO 9*)
• apply systematic listing strategies
• record describe and analyse the frequency of outcomes of probability experiments using frequency trees (ALSO 7*)
• enumerate sets and combinations of sets systematically, using tables, grids and Venn diagrams
• interpret, analyse and compare the distributions of data sets from univariate empirical distributions through appropriate graphical representation involving discrete, continuous and grouped data
• use and interpret scatter graphs of bivariate data
• recognise correlation
• draw estimated lines of best fit; make predictions
• know correlation does not indicate causation; interpolate and extrapolate apparent trends whilst knowing the dangers of so doing
• interpret, analyse and compare the distributions of data sets from univariate empirical distributions through appropriate measures of central tendency (median, mean, mode and modal class) and spread (range, including consideration of outliers)
• apply statistics to describe a population
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<tr><td>Understanding risk II</td><td>8</td></tr>
<tr><td>Presentation of data</td><td>8</td></tr>
<tr><td>Measuring data</td><td>8</td></tr>
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Numbers and the number system	Stage 9*	9 hours

KNOWLEDGE — **The Big Picture**: Number and Place Value progression map

- **use the concepts and vocabulary of prime numbers, highest common factor, lowest common multiple, prime factorisation, including using product notation and the unique factorisation theorem**
- **interpret standard form $A \times 10^n$, where $1 \le A < 10$ and n is an integer**
- **calculate with standard form $A \times 10^n$, where $1 \le A < 10$ and n is an integer (ALSO 9*)**

SKILLS

- Recall prime numbers up to 100
- Understand the meaning of prime factor
- Write a number as a product of its prime factors
- Use a Venn diagram to sort information
- Use prime factorisations to find the highest common factor of two numbers
- Use prime factorisations to find the lowest common multiple of two numbers
- Use a Venn diagram to find hcf and lcm of two numbers
- Know how to identify the first (any) significant figure in any number
- Know how to round to the first (any) significant figure in any number
- Review of the multiplication and division rule of indices
- Write a large (small) number in standard form
- Interpret a large (small) number written in standard form
- Enter a calculation written in standard form into a scientific calculator
- Interpret the standard form display of a scientific calculator
- Convert the answer from a standard form calculation back to a standard form answer
- Add (subtract) numbers written in standard form
- Multiply (divide) numbers written in standard form
- Calculate with negative indices in the context of standard form

Suggested resources

KM: Astronomical numbers
KM: Interesting standard form
KM: Powers of ten
KM: Maths to Infinity: Standard form
Powers of ten film (external site)
The scale of the universe animation (external site)

Learning review

- www.diagnosticquestions.com

Prerequisites

- Know the meaning of a prime number
- Recall prime numbers up to 50
- Understand the use of notation for powers
- Know how to round to the nearest whole number, 10, 100, 1000 and to decimal places
- Multiply and divide numbers by powers of 10

Mathematical language

Prime
Prime factor
Prime factorisation
Product
Venn diagram
Highest common factor
Lowest common multiple
Standard form
Significant figure

Notation

Index notation: e.g. 5^3 is read as ‘5 to the power of 3’ and means ‘3 lots of 5 multiplied together’
Standard form (see key concepts) is sometimes called ‘standard index form’, or more properly, ‘scientific notation’

Agreed common teaching approahes

Pupils should explore the ways to enter and interpret numbers in standard form on a scientific calculator. Different calculators may very well have different displays, notations and methods.
Liaise with the science department to establish when students first meet the use of standard form, and in what contexts they will be expected to interpret it.
NRICH: Divisibility testing
NCETM: Glossary

Common approaches

The following definition of a prime number should be used in order to minimise confusion about 1: A prime number is a number with exactly two factors.
The description ‘standard form’ is always used instead of ‘scientific notation’ or ‘standard index form’

Reasoning opportunities and probing questions

Cross Curricular Links

Possible misconceptions

• Show me two (three-digit) numbers with a highest common factor of 18. And another. And another… • Show me two numbers with a lowest common multiple of 240. And another. And another… • Jenny writes $7.1 \times 10^{-5} = 0.0000071$. Kenny writes $7.1 \times 10^{-5} = 0.000071$. Who do you agree with? Why?		• Many pupils believe that 1 is a prime number – a misconception which can arise if the definition is taken as 'a number which is divisible by itself and 1' • Some pupils may think 35 934 = 36 to two significant figures • When converting between ordinary and standard form some pupils may incorrectly connect the power to the number of zeros; e.g. $4 \times 10^5 = 400\,000$ so $4.2 \times 10^5 = 4\,200\,000$ • Similarly, when working with small numbers (negative powers of 10) some pupils may think that the power indicates how many zeros should be placed between the decimal point and the first non-zero digit

Calculating	Stage 9*	9 hours

KNOWLEDGE

The Big Picture: Calculation progression map

- **apply the four operations, including formal written methods, to integers, decimals and simple fractions (proper and improper), and mixed numbers – all both positive and negative**
- **use conventional notation for priority of operations, including brackets, powers, roots and reciprocals**
- **calculate with roots, and with integer indices**
- **use inequality notation to specify simple error intervals due to truncation or rounding**
- **apply and interpret limits of accuracy**

SKILLS

- Add or subtract from a negative number
- Add (or subtract) a negative number to (from) a positive number
- Add (or subtract) a negative number to (from) a negative number
- Multiply with negative numbers
- Divide with negative numbers
- Know how to square (or cube) a negative number, and understand why (at this level) you cannot square root a negative number
- Estimating the square root of a none square number
- Enter negative numbers into a calculator
- Interpret a calculator display when working with negative numbers
- Substitution and the application of the 4 operations and BIDMAS
- Understand how to use the order of operations including powers
- Understand how to use the order of operations including roots
- Calculate with positive indices (roots) using written methods
- Identify the minimum and maximum values of an amount that has been rounded (to nearest x, x d.p., x s.f.)
- Use inequalities to describe the range of values for a rounded value
- Solve problems involving the maximum and minimum values of an amount that has been rounded
- Introduce simple Upper / Lower Bound calculations

Suggested resources

KM: Summing up
KM: Developing negatives
KM: Sorting calculations
KM: Maths to Infinity: Directed numbers
Standards Unit: N9 Evaluating directed number statements
NRICH: Working with directed numbers

Learning review
www.diagnosticquestions.com

Prerequisites

- Fluently recall and apply multiplication facts up to 12 × 12
- Know and use column addition and subtraction
- Know the formal written method of long multiplication
- Know the formal written method of short division
- Convert between an improper fraction and a mixed number
- Know the order of operations for the four operations and brackets

Bring on the Maths^{+}: Moving on up!
Number and Place Value: v3

Mathematical language

Negative number
Directed number
Improper fraction
Top-heavy fraction
Mixed number
Operation
Inverse
Long multiplication
Short division
Power
Indices
Roots

Agreed common teaching approaches

Pupils need to know how to enter negative numbers into their calculator and how to interpret the display.
The grid method is promoted as a method that aids numerical understanding and later progresses to multiplying algebraic statements.
NRICH: Adding and subtracting positive and negative numbers
NRICH: History of negative numbers
NCETM: Departmental workshop: Operations with Directed Numbers
NCETM: Glossary

Common approaches
Teachers use the language 'negative number', and not 'minus number', to avoid confusion with calculations
Every classroom has a negative number washing line on the wall
Long multiplication and short division are to be promoted as the 'most efficient methods'.
If any acronym is promoted to help remember the order of operations, then BIDMAS is used as the I stands for indices.

Reasoning opportunities and probing questions

Cross Curricular Links

Possible misconceptions

• Convince me that -3 - -7 = 4 • Show me an example of a calculation involving addition of two negative numbers and the solution -10. And another. And another … • Create a Carroll diagram with ‘addition’, ‘subtraction’ as the column headings and ‘one negative number’, ‘two negative numbers’ as the row headings. Ask pupils to create (if possible) a calculation that can be placed in each of the four positions. If they think it is not possible, explain why. Repeat for multiplication and division.		• Some pupils may use a rule stated as ‘two minuses make a plus’ and make many mistakes as a result; e.g. -4 + -6 = 10 • Some pupils may incorrectly apply the principle of commutativity to subtraction; e.g. 4 – 7 = 3 • The order of operations is often not applied correctly when squaring negative numbers. As a result pupils may think that $x^2 = -9$ when x = -3. The fact that a calculator applies the correct order means that $-3^2 = -9$ and this can actually reinforce the misconception. In this situation brackets should be used as follows: $(-3)^2 = 9$.

Visualising and constructing	Stage 9*	*9 hours*

KNOWLEDGE

The Big Picture: Properties of Shape progression map

- **measure line segments and angles in geometric figures, including interpreting maps and scale drawings and use of bearings (ALSO 9*)**
- **interpret plans and elevations of 3D shapes (ALSO 7*)**
- **use scale factors, scale diagrams and maps**
- **identify, describe and construct similar shapes, including on coordinate axes, by considering enlargement**
- **use the standard ruler and compass constructions (perpendicular bisector of a line segment, constructing a perpendicular to a given line from/at a given point, bisecting a given angle)**

Return to overview

SKILLS	Suggested resources
• Use the concept of scaling in diagrams • Recap alternate, corresponding, interior and opposite angles • Measure and state a specified bearing • Construct a scale diagram involving bearings • Use bearings to solve geometrical problems • Understand the concept of back bearings • Know the vocabulary of enlargement • Find the centre of enlargement • Find the scale factor of an enlargement • Use the centre and scale factor to carry out an enlargement with positive and/or negative integer and fractional scale factors • Know and understand the vocabulary of plans and elevations • Interpret plans and elevations	KM: Outdoor Leisure 13 KM: Airports and hilltops KM: Plans and elevations KM: Transformation template KM: Enlargement I KM: Enlargement II KM: Investigating transformations with Autograph (enlargement and Main Event II). Dynamic example. WisWeb applet: Building houses NRICH: Who’s the fairest of them all? **Learning review** www.diagnosticquestions.com •

Prerequisites	Mathematical language	Agreed common teaching approahes
• Give an example of a shape and its enlargement (e.g. scale factor 2) with the guidelines drawn on. How many different ways can the scale factor be derived? • Show me an example of a sketch where the bearing of A from B is between 90° and 180°. • The bearing of A from B is ‘x’. Find the bearing of B from A in terms of ‘x’. Explain why this works. • Provide the plan and elevations of shapes made from some cubes. Challenge pupils to build the shape and place it in the correct orientation.	Compasses, Perpendicular bisector Locus, Loci, Arc, Plan, Perpendicular Elevation, Line segment, Bisect	Describing enlargement as a ‘scaling’ will help prevent confusion when dealing with fractional scale factors NCETM: Departmental workshops: Enlargement NCETM: Glossary **Common approaches** *All pupils should experience using dynamic software (e.g. Autograph) to visualise the effect of moving the centre of enlargement, and the effect of varying the scale factor.*

Reasoning opportunities and probing questions	Cross Curricular Links	Possible misconceptions

• Give an example of a shape and its enlargement (e.g. scale factor 2) with the guidelines drawn on. How many different ways can the scale factor be derived? • Show me an example of a sketch where the bearing of A from B is between 90° and 180°. • The bearing of A from B is 'x'. Find the bearing of B from A in terms of 'x'. Explain why this works. • Provide the plan and elevations of shapes made from some cubes. Challenge pupils to build the shape and place it in the correct orientation.		• Some pupils may think that the centre of enlargement always has to be (0,0), or that the centre of enlargement will be in the centre of the object shape. • If the bearing of A from B is 'x', then some pupils may think that the bearing of B from A is '180 – x'. • The north elevation is the view of a shape from the north (the north face of the shape), not the view of the shape while facing north.

Algebraic proficiency: tinkering	Stage 9*	9 hours

KNOWLEDGE

The Big Picture: Algebra progression map

- **use and interpret algebraic notation, including: a^2b in place of $a \times a \times b$, coefficients written as fractions rather than as decimals**
- **understand and use the concepts and vocabulary of factors**
- **rearrange formulae to change the subject**
- **simplify and manipulate algebraic expressions by expanding products of two binomials and factorising quadratic expressions of the form $x^2 + bx + c$**
- **argue mathematically to show algebraic expressions are equivalent, and use algebra to support and construct arguments (NEW)**
- **translate simple situations or procedures into algebraic expressions or formulae (NEW)**

SKILLS	Suggested resources
• Know how to write products algebraically • Use fractions when working in algebraic situations • Identify common factors (numerical and algebraic) of terms in an expression • Factorise an expression by taking out common factors • Simplify an expression involving terms with combinations of variables (e.g. $3a^2b + 4ab^2 + 2a^2 – a^2b$) • Know the multiplication (division, power, zero) law of indices • Understand that negative powers can arise • Substitute positive and negative numbers into formulae • Be aware of common scientific formulae • Know the meaning of the 'subject' of a formula • Change the subject of a formula when one step is required • Change the subject of a formula when two steps are required • Introduce changing the subject of a formula when two steps are required and factorisation is neccessary • Multiply two linear expressions of the form $(x + a)(x + b)$ • Multiply two linear expressions of the form $(x \pm a)(x \pm b)$ • Expand the expression $(x \pm a)^2$ • Introduce the difference of two squares • Introduce the expansion of 3 sets of brackets • Factorise a quadratic expression of the form $x^2 + bx + c$	KM: Missing powers KM: Laws of indices. Some useful questions. KM: Maths to Infinity: Indices NRICH: Temperature **Learning review** www.diagnosticquestions.com

Prerequisites	Mathematical language	Agreed common teaching approaches

• Know basic algebraic notation (the rules of algebra) • Simplify an expression by collecting like terms • Know how to multiply a single term over a bracket • Substitute positive numbers into expressions and formulae • Calculate with negative numbers	Product Variable Term Coefficient Common factor Factorise Power Indices Formula, Formulae Subject Change the subject **Notation** See key concepts above	During this unit pupils should experience factorising a quadratic expression such as $6x^2 + 2x$. Collaborate with the science department to establish a list of formulae that will be used, and ensure consistency of approach and experience. NCETM: Algebra NCETM: Departmental workshop: Index Numbers NCETM: Departmental workshops: Deriving and Rearranging Formulae NCETM: Glossary **Common approaches** *Once the laws of indices have been established, all teachers refer to 'like numbers multiplied, add the indices' and 'like numbers divided, subtract the indices. They also generalise to $a^m \times a^n = a^{m+n}$, etc.* *When changing the subject of a formula the principle of balancing (doing the same to both sides) must be used rather than a 'change side, change sign' approach.*
Reasoning opportunities and probing questions	**Cross Curricular Links**	**Possible misconceptions**
• Establish the multiplication, division and power laws of indices by writing products in full. Use the division law of indices to establish why $a^0 = 1$. • What is wrong with this statement and how can it be corrected: $5^2 \times 5^4 = 5^8$? • Jenny thinks that if y = 2x + 1 then x = (y – 1)/2. Kenny thinks that if y = 2x + 1 then x = y/2 – 1. Who do you agree with? Why?		• Some pupils may misapply the order of operation when changing the subject of a formula • Many pupils may think that $a^0 = 0$ • Some pupils may not consider 4ab and 3ba as 'like terms' and therefore will not 'collect' them when simplifying expressions

Exploring fractions, decimals and percentages	Stage 9*	*3 hours*

KNOWLEDGE — The Big Picture: Fractions, decimals and percentages progression map

- **work interchangeably with terminating decimals and their corresponding fractions (such as 3.5 and 7/2 or 0.375 or 3/8) (ALSO 7*)**

SKILLS	Suggested resources
• Write a decimal as a fraction • Write a fraction as a decimal • Write a decimal as a percentage • Write a percentage as a decimal • Write a fraction as a percentage • Write a percentage as a fraction • Identify if a fraction is terminating or recurring • Convert a recurring decimal into a fraction • Recall some decimal and fraction equivalents (e.g. tenths, fifths, eighths) • Write a fraction in its lowest terms by cancelling common factors • Identify when a fraction can be scaled to tenths or hundredths • Convert a fraction to a decimal by scaling (when possible) • Use a calculator to change any fraction to a decimal and all the other conversions	KM: FDP conversion. Templates for taking notes. KM: Fraction sort. Tasks one and two only. KM: Maths to Infinity: Fractions, decimals, percentages, ratio, proportion NRICH: Matching fractions, decimals and percentages **Learning review** www.diagnosticquestions.com

Prerequisites	Mathematical language	Agreed common teaching approahes

• Understand that fractions, decimals and percentages are different ways of representing the same proportion • Convert between mixed numbers and top-heavy fractions • Write one quantity as a fraction of another	Fraction Mixed number Top-heavy fraction Percentage Decimal Proportion Terminating Recurring Simplify, Cancel **Notation** Diagonal and horizontal fraction bar	The diagonal fraction bar (solidus) was first used by Thomas Twining (1718) when recorded quantities of tea. The division symbol (÷) is called an obelus, but there is no name for a horizontal fraction bar. NRICH: History of fractions NRICH: Teaching fractions with understanding NCETM: Glossary **Common approaches** *All pupils should use the horizontal fraction bar to avoid confusion when fractions are coefficients in algebraic situations*
Reasoning opportunities and probing questions	**Cross Curricular Links**	**Possible misconceptions**
• Without using a calculator, convince me that 3/8 = 0.375 • Show me a fraction / decimal / percentage equivalent. And another. And another … • What is the same and what is different: 2.5, 25%, 0.025 ?		• Some pupils may make incorrect links between fractions and decimals such as thinking that 1/5 = 0.15 • Some pupils may think that 5% = 0.5, 4% = 0.4, etc. • Some pupils may think it is not possible to have a percentage greater than 100%.

Proportional reasoning | Stage 9* | 9 hours

KNOWLEDGE

The Big Picture: Ratio and Proportion progression map

- **express the division of a quantity into two parts as a ratio; apply ratio to real contexts and problems (such as those involving conversion, comparison, scaling, mixing, concentrations)**
- **understand and use proportion as equality of ratios**
- **express a multiplicative relationship between two quantities as a ratio or a fraction**
- **relate ratios to fractions and to linear functions**
- **use compound units (such as speed, rates of pay, unit pricing, density and pressure)**
- **change freely between compound units (e.g. speed, rates of pay, prices) in numerical contexts**
-

SKILLS

- Identify ratio in a real-life context
- Write a ratio to describe a situation
- Identify proportion in a situation
- Find a relevant multiplier in a situation involving proportion
- Use fractions fluently in situations involving ratio or proportion
- Understand the connections between ratios and fractions
- Understand the meaning of a compound unit
- Know the connection between speed, distance and time
- Solve problems involving speed
- Identify when it is necessary to convert quantities in order to use a sensible unit of measure
- Know the difference between direct and inverse proportion
- Recognise direct (inverse) proportion in a situation
- Know the features of a graph that represents a direct (inverse) proportion situation
- Know the features of an expression (or formula) that represents a direct (inverse) proportion situation
- Understand why speed, density and pressure are known as compound units
- Know the definition of density (pressure, population density, speed)
- Solve problems involving density (pressure, speed)
- Convert between units of density

Suggested resources

KM: Proportion for real
KM: Investigating proportionality
KM: Maths to Infinity: Fractions, decimals, percentages, ratio, proportion
NRICH: In proportion
Standards Unit: N6 Developing proportional reasoning

Learning review
www.diagnosticquestions.com

Prerequisites

- Understand and use ratio notation
- Divide an amount in a given ratio

Mathematical language

Ratio
Proportion
Proportional
Multiplier
Speed
Unitary method
Units
Compound unit

Notation
Kilometres per hour is written as km/h or kmh^{-1}
Metres per second is written as m/s or ms^{-1}

Agreed common teaching approahes

NRICH: Ratio or proportion?
NRICH: Roasting old chestnuts 3
NCETM: The Bar Model
NCETM: Multiplicative reasoning
NCETM: Departmental workshops: Proportional Reasoning
NCETM: Glossary

Common approaches
All pupils are taught to set up a 'proportion table' and use it to find the multiplier in situations involving proportion

Reasoning opportunities and probing questions

- Show me an example of two quantities that will be in proportion. And another. And another …
- (Showing a table of values such as the one below) convince me that this information shows a proportional relationship

6	9
10	15
14	21

- Which is the faster speed: 60 km/h or 10 m/s? Explain why.

Cross Curricular Links

Possible misconceptions

- Many pupils will want to identify an additive relationship between two quantities that are in proportion and apply this to other quantities in order to find missing amounts
- Some pupils may think that a multiplier always has to be greater than 1
- When converting between times and units, some pupils may base their working on 100 minutes = 1 hour

Pattern sniffing	Stage 9*	3 hours

KNOWLEDGE

The Big Picture: Algebra progression map

- **generate terms of a sequence from either a term-to-term or a position-to-term rule (ALSO 7*)**
- **deduce expressions to calculate the nth term of linear sequences (ALSO 7*)**
- **recognise and use Fibonacci type sequences, quadratic sequences**

Return to overview

SKILLS

- Generate a sequence from a term-to-term rule
- Understand the meaning of a position-to-term rule
- Use a position-to-term rule to generate a sequence
- Find the position-to-term rule for a given sequence
- Use algebra to describe the position-to-term rule of a linear sequence (the nth term)
- Use the nth term of a sequence to deduce if a given number is in a sequence
- Generate a sequence using a spreadsheet
- Generate terms of a quadratic sequence
- Identify quadratic sequences
- Establish the first and second differences of a quadratic sequence
- Find the next three terms in any quadratic sequence
- Find the term in x^2 for a quadratic sequence

Suggested resources

KM: Spreadsheet sequences
KM: Generating sequences
KM: Maths to Infinity: Sequences
KM: Stick on the Maths: Linear sequences
NRICH: Charlie's delightful machine
NRICH: A little light thinking
NRICH: Go forth and generalise

Learning review
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Prerequisites

- Use a term-to-term rule to generate a sequence
- Find the term-to-term rule for a sequence
- Describe a sequence using the term-to-term rule

Mathematical language

Sequence
Linear
Term
Difference
Term-to-term rule
Position-to-term rule
Ascending
Descending

Notation
T(n) is often used when finding the nth term of sequence

Agreed common teaching approahes

Using the nth term for times tables is a powerful way of finding the nth term for any linear sequence. For example, if the pupils understand the 3 times table can ne described as '3n' then the linear sequence 4, 7, 10, 13, … can be described as the 3 times table 'shifted up' one place, hence 3n + 1.
Exploring statements such as 'is 171 is in the sequence 3, 9, 15, 21, 27, ..?' is a very powerful way for pupils to realise that 'term-to-term' rules can be inefficient and therefore 'position-to-term' rules (nth term) are needed.
NCETM: Algebra
NCETM: Glossary

Common approaches
Teachers refer to a sequence such as 2, 5, 8, 11, … as 'the three times table minus one', to help pupils construct their understanding of the nth term of a sequence.
All students have the opportunity to use spreadsheets to generate sequences

Reasoning opportunities and probing questions

- Show me a sequence that could be generated using the nth term 4n ± c. And another. And another …
- What's the same, what's different: 4, 7, 10, 13, 16, …. , 2, 5, 8, 11, 14, … , 4, 9, 14, 19, 24, …. and 4, 10, 16, 22, 28, …?
- The 4th term of a linear sequence is 15. Show me the nth term of a sequence with this property. And another. And another …
- Convince me that the nth term of the sequence 2, 5, 8, 11, … is 3n -1 .
- Kenny says the 171 is in the sequence 3, 9, 15, 21, 27, … Do you agree with Kenny? Explain your reasoning.

Cross Curricular Links

Possible misconceptions

- Some pupils will think that the nth term of the sequence 2, 5, 8, 11, … is n + 3.
- Some pupils may think that the (2n)th term is double the nth term of a linear sequence.
- Some pupils may think that sequences with nth term of the form 'ax ± b' must start with 'a'.

Investigating angles	Stage 9*	*6 hours*

KNOWLEDGE

The Big Picture: Position and direction progression map

- **derive and use the sum of angles in a triangle (e.g. to deduce and use the angle sum in any polygon, and to derive properties of regular polygons)**
- **use the basic congruence criteria for triangles (SSS, SAS, ASA, RHS) (ALSO 9*)**

Return to overview

SKILLS

- Identify alternate angles and know that they are equal
- Identify corresponding angles and know that they are equal
- Use knowledge of alternate and corresponding angles to calculate missing angles in geometrical diagrams
- Establish the fact that angles in a triangle must total 180°
- Use the fact that angles in a triangle total 180° to work out the total of the angles in any polygon
- Establish the size of an interior angle in a regular polygon
- Know the total of the exterior angles in any polygon
- Establish the size of an exterior angle in a regular polygon
- Know the criteria for triangles to be congruent (SSS, SAS, ASA, RHS)
- Identify congruent triangles

Suggested resources

KM: Alternate and corresponding angles
KM: Perplexing parallels
KM: Investigating polygons
KM: Maths to Infinity: Lines and angles
KM: Stick on the Maths: Alternate and corresponding angles
KM: Stick on the Maths: Geometrical problems
NRICH: Ratty

Learning review
www.diagnosticquestions.com

Prerequisites

- Use angles at a point, angles at a point on a line and vertically opposite angles to calculate missing angles in geometrical diagrams
- Know that the angles in a triangle total 180°

Mathematical language

Degrees
Right angle, acute angle, obtuse angle, reflex angle
Vertically opposite
Geometry, geometrical
Parallel
Alternate angles, corresponding angles
Interior angle, exterior angle
Regular polygon

Notation
Dash notation to represent equal lengths in shapes and geometric diagrams
Arrow notation to show parallel lines

Agreed common teaching approahes

The KM: Perplexing parallels resource is a great way for pupils to discover practically the facts for alternate and corresponding angles.
Pupils have established the fact that angles in a triangle total 180° in Stage 7. However, using alternate angles they are now able to prove this fact.
Encourage pupils to draw regular and irregular convex polygons to discover the sum of the interior angles = (n – 2) × 180°.
NCETM: Glossary

Common approaches
Teachers insist on correct mathematical language (and not F-angles or Z-angles for example)

Reasoning opportunities and probing questions

- Show me a pair of alternate (corresponding) angles. And another. And another …
- Jenny thinks that hexagons are the only polygon that tessellates. Do you agree? Explain your reasoning.
- Convince me that the angles in a triangle total 180°.
- Convince me that the interior angle of a pentagon is 540°.
- Always/ Sometimes/ Never: The sum of the interior angles of an n-sided polygon can be calculated using sum = (n – 2) × 180°.
- Always/ Sometimes/ Never: The sum of the exterior angles of a polygon is 360°.

Cross Curricular Links

Possible misconceptions

- Some pupils may think that alternate and/or corresponding angles have a total of 180° rather than being equal.
- Some pupils may think that the sum of the interior angles of an n-sided polygon can be calculated using Sum = n × 180°.
- Some pupils may think that the sum of the exterior angles increases as the number of sides of the polygon increases.

Calculating fractions, decimals and percentages	Stage 9*	6 hours

KNOWLEDGE

The Big Picture: Fractions, decimals and percentages progression map

- **interpret fractions and percentages as operators**
- **work with percentages greater than 100%**
- **solve problems involving percentage change, including original value problems, and simple interest including in financial mathematics (ALSO 7*)**
- **calculate exactly with fractions**

Return to overview

SKILLS

- Recognise when a fraction (percentage) should be interpreted as a number
- Recognise when a fraction (percentage) should be interpreted as a operator
- Identify the multiplier for a percentage increase or decrease when the percentage is greater than 100%
- Use calculators to increase an amount by a percentage greater than 100%
- Solve problems involving percentage change
- Solve original value problems when working with percentages
- Solve financial problems including simple interest
- Understand the meaning of giving an exact solution
- Solve problems that require exact calculation with fractions

Suggested resources

KM: Stick on the Maths: Proportional reasoning
KM: Stick on the Maths: Multiplicative methods
KM: Percentage identifying
NRICH: One or both
NRICH: Antiques roadshow

Learning review
www.diagnosticquestions.com

Prerequisites

- Apply the four operations to proper fractions, improper fractions and mixed numbers
- Use calculators to find a percentage of an amount using multiplicative methods
- Identify the multiplier for a percentage increase or decrease
- Use calculators to increase (decrease) an amount by a percentage using multiplicative methods
- Know that percentage change = actual change ÷ original amount

Mathematical language

Proper fraction, improper fraction, mixed number
Simplify, cancel, lowest terms
Percent, percentage
Percentage change
Original amount
Multiplier
(Simple) interest
Exact

Notation
Mixed number notation
Horizontal / diagonal bar for fractions

Agreed common teaching approahes

The bar model is a powerful strategy for pupils to 're-present' a problem involving percentage change.
Only simple interest should be explored in this unit. Compound interest will be developed later.
NCETM: The Bar Model
NCETM: Glossary

Common approaches
When adding and subtracting mixed numbers pupils are taught to convert to improper fractions as a general strategy
Teachers use the horizontal fraction bar notation at all times

Reasoning opportunities and probing questions

- Convince me that the multiplier for a 150% increase is 2.5
- Kenny buys a poncho in a 25% sale. The sale price is £40. Kenny thinks that the original is £50. Do you agree with Kenny? Explain your answer.
- Jenny thinks that increasing an amount by 200% is the same as multiplying by 3. Do you agree with Jenny? Explain your answer.

Cross Curricular Links

Agreed common teaching approahes

- Some pupils may think that the multiplier for a 150% increase is 1.5
- Some pupils may think that increasing an amount by 200% is the same as doubling.
- In isolation, pupils may be able to solve original value problems confidently. However, when it is necessary to identify the type of percentage problem, many pupils will apply a method for a more simple percentage increase / decrease problem. If pupils use models (e.g. the bar model, or proportion tables) to represent all problems then they are less likely to make errors in identifying the type of problem.

Solving equations and inequalities	Stage 9*	3 hours

KNOWLEDGE — The Big Picture: Algebra progression map

- **solve linear equations with the unknown on both sides of the equation (ALSO, 7*)**
- **find approximate solutions to linear equations using a graph**
- **solve, in simple cases, two linear simultaneous equations in two variables algebraically (ALSO, 9*)**
- **derive an equation (or two simultaneous equations), solve the equation(s) and interpret the solution**
- **find approximate solutions to simultaneous equations using a graph**
- **solve linear inequalities in two variables (NEW)**

Return to overview

SKILLS

- Identify the correct order of undoing the operations in an equation
- Solve linear equations with the unknown on one side when the solution is a negative number
- Solve linear equations with the unknown on both sides when the solution is a whole number
- Solve linear equations with the unknown on both sides when the solution is a fraction
- Solve linear equations with the unknown on both sides when the solution is a negative number
- Solve linear equations with the unknown on both sides when the equation involves brackets
- Recognise that the point of intersection of two graphs corresponds to the solution of a connected equation
- Check the solution to an equation by substitution
- Understand the concept of simultaneous equations
- Find approximate solutions to simultaneous equations using a graph
- Understand the concept of solving simultaneous equations by elimination*
- Solve two linear simultaneous equations in two variables in very simple cases (no multiplication required)
- Solve two linear simultaneous equations in two variables in simple cases (multiplication of one equation only required)
- Derive and solve two simultaneous equations

Suggested resources

KM: Solving equations
KM: Stick on the Maths: Constructing and solving equations
NRICH: Think of Two Numbers

Learning review
www.diagnosticquestions.com

Prerequisites

- Choose the required inverse operation when solving an equation
- Solve linear equations by balancing when the solution is a whole number or a fraction

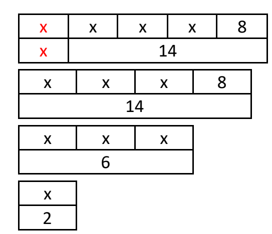
Mathematical language

Algebra, algebraic, algebraically
Unknown
Equation
Operation
Solve
Solution
Brackets
Symbol
Substitute
Graph
Point of intersection

Notation
The lower case and upper case of a letter should not be used interchangeably when worked with algebra
Juxtaposition is used in place of '×'. 2a is used rather than a2.
Division is written as a fraction

Agreed common teaching approahes

This unit builds on the wok solving linear equations with unknowns on one side in Stage 7. It is essential that pupils are secure with solving these equations before moving onto unknowns on both sides.
Encourage pupils to 're-present' the problem using the Bar Model.
NCETM: The Bar Model
NCETM: Algebra
NCETM: Glossary



Common approaches
All pupils should solve equations by balancing:

$$
\begin{aligned}
4x + 8 &= 14 + x \\
-x \quad & \quad -x \\
3x + 8 &= 14 \\
-8 \quad & \quad -8 \\
3x &= 6 \\
\div 3 \quad & \quad \div 3 \\
x &= 2
\end{aligned}
$$

Reasoning opportunities and probing questions

Cross Curricular Links

Possible misconceptions

• Show me an (one-step, two-step) equation with a solution of -8 (negative, fractional solution). And another. And another … • Show me a two-step equation that is 'easy' to solve. And another. And another … • What's the same, what's different: 2x + 7 = 25, 3x + 7 = x + 25, x + 7 = 7 – x, 4x + 14 = 50 ? • Convince me how you could use graphs to find solutions, or estimates, for equations.		• Some pupils may think that you always have to manipulate the equation to have the unknowns on the LHS of the equal sign, for example 2x – 3 = 6x + 6 • Some pupils think if 4x = 2 then x = 2. • When solving equations of the form 2x – 8 = 4 – x, some pupils may subtract 'x' from both sides.

Calculating space | Stage 9* | 9 hours

KNOWLEDGE

The Big Picture: Measurement and mensuration progression map

- **compare lengths, areas and volumes using ratio notation**
- **calculate perimeters of 2D shapes, including circles**
- **know the formulae: circumference of a circle = 2πr = πd, area of a circle = πr²**
- **calculate areas of circles and composite shapes**
- **know the formulae for: Pythagoras' theorem, a² + b² = c², and apply it to find lengths in right-angled triangles in two dimensional figures**
- **know and apply formulae to calculate volume of right prisms (including cylinders)**
- **identify and apply circle definitions and properties, including: tangent, arc, sector and segment**

SKILLS

- Know the vocabulary of circles
- Know that the number π (pi) = 3.1415926535…
- Recall π to two decimal places
- Know the formula circumference of a circle = 2πr = πd
- Calculate the circumference of a circle when radius (diameter) is given
- Calculate the radius (diameter) of a circle when the circumference is known
- Calculate the perimeter of composite shapes that include sections of a circle
- Know the formula area of a circle = πr²
- Calculate the area of a circle when radius (diameter) is given
- Calculate the radius (diameter) of a circle when the area is known
- Calculate the area of composite shapes that include sections of a circle
- Know the formula for finding the volume of a right prism (cylinder)
- Calculate the volume of a right prism (cylinder)
- Know the vocabulary of circles
- Know how to find arc length
- Calculate the arc length of a sector when radius is given
- Know how to find the area of a sector
- Calculate the area of a sector when radius is given
- Calculate the angle of a sector when the arc length and radius are known
- Know how to find the surface area of a right prism (cylinder)
- Calculate the surface area of a right prism (cylinder)

Suggested resources

KM: Circle connections
KM: Circle problems
KM: Maths to Infinity: Area and Volume
KM: Stick on the Maths: Circumference and area of a circle
KM: Stick on the Maths: Right prisms
NRICH: Blue and White
NRICH: Efficient Cutting
NRICH: Cola Can

Learning review
www.diagnosticquestions.com

Prerequisites

- Know how to use formulae to find the area of rectangles, parallelograms, triangles and trapezia
- Know how to find the area of compound shapes

Mathematical language

Circle
Centre
Radius, diameter, chord, circumference
Pi
(Right) prism
Cross-section
Cylinder
Polygon, polygonal
Solid

Notation
π
Abbreviations of units in the metric system: km, m, cm, mm, mm², cm², m², km², mm³, cm³, km³

Agreed common teaching approahes

C = πd can be established by investigating the ratio of the circumference to the diameter of circular objects (wheel, clock, tins, glue sticks, etc.)
Pupils need to understand this formula in order to derive A = πr².
A prism is a solid with constant polygonal cross-section. A right prism is a prism with a cross-section that is perpendicular to the 'length'.
NCETM: Glossary

Common approaches
The area of a circle is derived by cutting a circle into many identical sectors and approximating a parallelogram
Every classroom has a set of area posters on the wall
The formula for the volume of a prism is 'area of cross-section × length' even if the orientation of the solid suggests that height is required
Pupils use area of a trapezium = $\frac{(a+b)h}{2}$ and area of a triangle = area = $\frac{bh}{2}$

Reasoning opportunities and probing questions

- Convince me C = 2πr = πd.
- What is wrong with this statement? How can you correct it? The area of a circle with radius 7 cm is approximately 441 cm² because (3 × 7)² = 441.
- Convince me the area of a semi-circle = $\frac{\pi d^2}{4}$
- Name a right prism. And another. And another …
- Convince me that a cylinder is not a prism

Cross Curricular Links

Possible misconceptions

- Some pupils will work out (π × radius)² when finding the area of a circle
- Some pupils may use the sloping height when finding cross-sectional areas that are parallelograms, triangles or trapezia
- Some pupils may think that the area of a triangle = base × height
- Some pupils may think that you multiply all the numbers to find the volume of a prism
- Some pupils may confuse the concepts of surface area and volume

Algebraic proficiency: visualising | Stage 9* | 9 hours

KNOWLEDGE

The Big Picture: Algebra progression map

- **identify and interpret gradients and intercepts of linear functions graphically and algebraically**
- **recognise, sketch and interpret graphs of linear functions and simple quadratic functions**
- **plot and interpret graphs and graphs of non-standard (*piece-wise linear)* functions in real contexts, to find approximate solutions to problems such as simple kinematic problems involving distance and speed**
- **use the form *y = mx + c* to identify parallel lines**
- **interpret the gradient of a straight line graph as a rate of change**

Return to overview

SKILLS

- Know that graphs of functions of the form y = mx + c, x ± y = c and ax ± by = c are linear
- Plot graphs of functions of the form y = mx + c (x ± y = c, ax ± by = c)
- Understand the concept of the gradient of a straight line
- Find the gradient of a straight line on a unit grid
- Find the y-intercept of a straight line
- Sketch a linear graph
- Distinguish between a linear and quadratic graph
- Plot graphs of quadratic functions of the form y = x^2 ± c
- Sketch a simple quadratic graphPlot and interpret linear graphs
- Plot and quadratic graphs
- Model real situations using linear graphs
- Plot and interpret graphs of piece-wise linear functions in real contexts
- Plot and interpret distance-time graphs (speed-time graphs)
- Find approximate solutions to kinematic problems involving distance and speed
- Use the form y = mx + c to identify parallel lines
- Rearrange an equation into the form y = mx + c
- Find the equation of a line through one point with a given gradient
- Find the equation of a line through two given points
- Interpret the gradient of a straight line graph as a rate of change

Suggested resources

KM: Matching graphs
KM: Autograph 1
KM: Autograph 2
KM: The hare and the tortoise

Learning review
KM: 8M11 BAM Task

Prerequisites

- Use coordinates in all four quadrants
- Write the equation of a line parallel to the x-axis or the y-axis
- Draw a line parallel to the x-axis or the y-axis given its equation
- Identify the lines y = x and y = -x
- Draw the lines y = x and y = -x
- Substitute positive and negative numbers into formulae

Mathematical language

Plot
Equation (of a graph)
Function
Formula
Linear
Coordinate plane
Gradient
y-intercept
Substitute
Quadratic
Piece-wise linear
Model
Kinematic, Speed, Distance

Notation
y = mx + c

Agreed common teaching approaches

When plotting graphs of functions of the form y = mx + c a table of values can be useful. Note that negative number inputs can cause difficulties. Pupils should be aware that the values they have found for linear functions should correspond to a straight line.
NCETM: Glossary

Common approaches
Pupils are taught to use positive numbers wherever possible to reduce potential difficulties with substitution of negative numbers
Students plot points with a 'x' and not '•'
Students draw graphs in pencil
All pupils use dynamic geometry software to explore graphs of functions

Reasoning opportunities and probing questions

- Draw a distance-time graph of your journey to school. Explain the key features.
- Show me a point on this line (e.g. y = 2x + 1). And another, and another …
- (Given an appropriate distance-time graph) convince me that Kenny is stationary between 10: 00 a.m. and 10:45 a.m.

Cross Curricular Links

Possible misconceptions

- When plotting linear graphs some pupils may draw a line segment that stops at the two most extreme points plotted
- Some pupils may think that a sketch is a very rough drawing. It should still identify key features, and look neat, but will not be drawn to scale
- Some pupils may think that a positive gradient on a distance-time graph corresponds to a section of the journey that is uphill
- Some pupils may think that the graph y = x^2 + c is the graph of y = x^2 translated horizontally.

Understanding risk II	Stage 9*	6 hours

KNOWLEDGE

The Big Picture: Probability progression map

- **enumerate sets and combinations of sets systematically, using tree diagrams (ALSO 9*)**
- **apply systematic listing strategies**
- **record describe and analyse the frequency of outcomes of probability experiments using frequency trees (ALSO 7*)**
- **enumerate sets and combinations of sets systematically, using tables, grids and Venn diagrams**

Return to overview

SKILLS

- List outcomes of combined events using a tree diagram
- Label a tree diagram with probabilities
- Label a tree diagram with probabilities when events are dependent
- Use a tree diagram to calculate probabilities of independent combined events
- Use a tree diagram to calculate probabilities of dependent combined events
- List all elements in a combination of sets using a Venn diagram
- List outcomes of an event systematically
- Use a table to list all outcomes of an event
- List outcomes of an event using a grid (two-way table)
- Use frequency trees to record outcomes of probability experiments
- Make conclusions about probabilities based on frequency trees
- Construct theoretical possibility spaces for combined experiments with equally likely outcomes
- Calculate probabilities using a possibility space
- Use theoretical probability to calculate expected outcomes
- Use experimental probability to calculate expected outcomes

Suggested resources

KM: Sample spaces
KM: Race game

Learning review=
www.diagnosticquestions.com

Prerequisites

- Convert between fractions, decimals and percentages
- Understand the use of the 0-1 scale to measure probability
- Work out theoretical probabilities for events with equally likely outcomes
- Know how to represent a probability
- Know that the sum of probabilities for all outcomes is 1

Mathematical language

Outcome
Event
Experiment, Combined experiment
Frequency tree
Enumerate
Set
Venn diagram
Possibility space, sample space
Equally likely outcomes
Theoretical probability
Random
Bias, Fairness
Relative frequency

Notation
P(A) for the probability of event A
Probabilities are expressed as fractions, decimals or percentage. They should not be expressed as ratios (which represent odds) or as words

Agreed common teaching approahes

The Venn diagram was invented by John Venn (1834 – 1923)
NCETM: Glossary

Common approaches
All students are taught to use 'DIME' probability recording charts
All classes carry out the 'race game' as a simulated horse race with horses numbered 1 to 12

Reasoning opportunities and probing questions

- Show me a way of listing all outcomes when two coins are flipped
- Convince me that there are more than 12 outcomes when two six-sided dice are rolled
- Convince me that 7 is the most likely total when two dice are rolled

Cross Curricular Links

Possible misconceptions

- Some students may think that there are only three outcomes when two coins are flipped, or that there are only six outcomes when three coins are flipped
- Some students may think that there are 12 unique outcomes when two dice are rolled
- Some students may think that there are 12 possible totals when two dice are rolled

Presentation of data	Stage 9*	6 hours

KNOWLEDGE

The Big Picture: Statistics progression map

- **interpret, analyse and compare the distributions of data sets from univariate empirical distributions through appropriate graphical representation involving discrete, continuous and grouped data**
- **use and interpret scatter graphs of bivariate data**
- **recognise correlation**
- **draw estimated lines of best fit; make predictions**
- **know correlation does not indicate causation; interpolate and extrapolate apparent trends whilst knowing the dangers of so doing**

Return to overview

SKILLS

- Know the meaning of continuous data
- Interpret a grouped frequency table for continuous data
- Construct a grouped frequency table for continuous data
- Construct histograms for grouped data with equal class intervals
- Interpret histograms for grouped data with equal class intervals
- Construct and use the horizontal axis of a histogram correctly
- Plot a scatter diagram of bivariate data
- Understand the meaning of 'correlation'
- Interpret a scatter diagram using understanding of correlation
- Understand that correlation does not indicate causation
- Interpret a scatter diagram using understanding of correlation
- Construct a line of best fit on a scatter diagram
- Use a line of best fit to estimate values
- Know when it is appropriate to use a line of best fit to estimate values

Suggested resources

KM: Make a 'human' scatter graph by asking pupils to stand at different points on a giant set of axes.
KM: Gathering data
KM: Spreadsheet statistics
KM: Stick on the Maths HD2: Selecting and constructing graphs and charts
KM: Stick on the Maths HD3: Working with grouped data

Learning review

- www.diagnosticquestions.com

Prerequisites

- Know the meaning of discrete data
- Interpret and construct frequency tables
- Construct and interpret pictograms, bar charts, pie charts, tables and vertical line charts

Mathematical language

Data
Categorical data, Discrete data
Continuous data, Grouped data
Table, Frequency table
Frequency
Histogram
Scale, Graph
Axis, axes
Scatter graph (scatter diagram, scattergram, scatter plot)
Bivariate data
(Linear) Correlation
Positive correlation, Negative correlation

Notation

Correct use of inequality symbols when labeling groups in a frequency table

Agreed common teaching approahes

The word histogram is often misused and an internet search of the word will usually reveal a majority of non-histograms. The correct definition is 'a diagram made of rectangles whose areas are proportional to the frequency of the group'. If the class widths are equal, as they are in this unit of work, then the vertical axis shows the frequency. It is only later that pupils need to be introduced to unequal class widths and frequency density.
Lines of best fit on scatter diagrams are not introduced until Stage 9, although pupils may well have encountered both lines and curves of best fit in science by this time.
NCETM: Glossary

Common approaches

All students collect data about their class's height and armspan when first constructing a scatter diagram

Reasoning opportunities and probing questions

- Show me a scatter graph with positive (negative, no) correlation. And another. And another.
- Show me a histogram. And another. And another.
- Kenny thinks that histogram is just a 'fancy' name for a bar chart. Do you agree with Kenny? Explain your answer.
- What's the same and what's different: histogram, scatter diagram, bar chart, pie chart?
- Always/Sometimes/Never: A scatter graph

Cross Curricular Links

Possible misconceptions

- Some pupils may label the bar of a histogram rather than the boundaries of the bars
- Some pupils may leave gaps between the bars in a histogram
- Some pupils may misuse the inequality symbols when working with a grouped frequency table

Measuring data	Stage 9*	6 hours

KNOWLEDGE

The Big Picture: Statistics progression map

- **interpret, analyse and compare the distributions of data sets from univariate empirical distributions through appropriate measures of central tendency (median, mean, mode and modal class) and spread (range, including consideration of outliers)**
- **apply statistics to describe a population**

Return to overview

SKILLS

- Find the modal class of set of grouped data
- Find the class containing the median of a set of data
- Find the midpoint of a class
- Calculate an estimate of the mean from a grouped frequency table
- Estimate the range from a grouped frequency table
- Analyse and compare sets of data
- Appreciate the limitations of different statistics (mean, median, mode, range)
- Choose appropriate statistics to describe a set of data
- Justify choice of statistics to describe a set of data
- Investigate averages

Suggested resources

KM: Swillions
KM: Lottery project
NRICH: Half a Minute

Learning review
www.diagnosticquestions.com

Prerequisites

- Understand the mean, mode and median as measures of typicality (or location)
- Find the mean, median, mode and range of a set of data
- Find the mean, median, mode and range from a frequency table

Mathematical language

Average
Spread
Consistency
Mean
Median
Mode
Range
Statistic
Statistics
Approximate, Round
Calculate an estimate
Grouped frequency
Midpoint

Notation
Correct use of inequality symbols when labeling groups in a frequency table

Agreed common teaching approahes

The word 'average' is often used synonymously with the mean, but it is only one type of average. In fact, there are several different types of mean (the one in this unit properly being named as the 'arithmetic mean').
NCETM: Glossary

Common approaches
Every classroom has a set of statistics posters on the wall
All students are taught to use mathematical presentation correctly when calculating and rounding solutions, e.g. (21 + 56 + 35 + 12) ÷ 30 = 124 ÷ 30 = 41.3 to 1 d.p.

Reasoning opportunities and probing questions

- Show me an example of an outlier. And another. And another.
- Convince me why the mean from a grouped set of data is only an estimate.
- What's the same and what's different: mean, modal class, median, range?
- Always/Sometimes/Never: A set of grouped data will have one modal class
- Convince me how to estimate the range for grouped data.

Cross Curricular Links

Possible misconceptions

- Some pupils may incorrectly estimate the mean by dividing the total by the numbers of groups rather than the total frequency.
- Some pupils may incorrectly think that there can only be one model class.
- Some pupils may incorrectly estimate the range of grouped data by subtracting the upper bound of the first group from the lower bound of the last group.