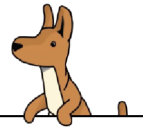
# Mathematics overview: Stage 8

| *Unit* | *Hours* | *KNOWLEDGE* |
|---|---|---|
| Numbers and the number system | 12 | • interpret standard form A × 10n, where 1 ≤ A < 10 and n is an integer<br>• use inequality notation to specify simple error intervals due to truncation or rounding (NEW)<br>• apply and interpret limits of accuracy (NEW)<br>• apply the four operations, including formal written methods, to integers, decimals and simple fractions (proper and improper), and mixed numbers – all both positive and negative<br>• use conventional notation for priority of operations, including brackets, powers, roots and reciprocals<br>• calculate with roots, and with integer indices (NEW)<br>• calculate exactly with fractions<br>• measure line segments and angles in geometric figures, including interpreting maps and scale drawings and use of bearings<br>• identify, describe and construct similar shapes, including on coordinate axes, by considering enlargement<br>• interpret plans and elevations of 3D shapes<br>• use scale factors, scale diagrams and maps<br>• use and interpret algebraic notation, including: $a^2b$ in place of a × a × b, coefficients written as fractions rather than as decimals<br>• understand and use the concepts and vocabulary of factors<br>• simplify and manipulate algebraic expressions by taking out common factors and simplifying expressions involving sums, products and powers, including the laws of indices<br>• substitute numerical values into scientific formulae<br>• work interchangeably with terminating decimals and their corresponding fractions (such as 3.5 and 7/2 or 0.375 or 3/8)<br>• express the division of a quantity into two parts as a ratio; apply ratio to real contexts and problems (such as those involving conversion, comparison, scaling, mixing, concentrations)<br>• identify and work with fractions in ratio problems<br>• understand and use proportion as equality of ratios<br>• express a multiplicative relationship between two quantities as a ratio or a fraction<br>• use compound units (such as speed, rates of pay, unit pricing)<br>• change freely between compound units (e.g. speed, rates of pay, prices) in numerical contexts<br>• relate ratios to fractions and to linear functions<br>• generate terms of a sequence from either a term-to-term or a position-to-term rule<br>• deduce expressions to calculate the nth term of linear sequences (ALSO 7)<br>• understand and use alternate and corresponding angles on parallel lines<br>• derive and use the sum of angles in a triangle (e.g. to deduce and use the angle sum in any polygon, and to derive properties of regular polygons)<br>• interpret fractions and percentages as operators<br>• work with percentages greater than 100%<br>• solve problems involving percentage change, including original value problems, and simple interest including in financial mathematics<br>• solve linear equations with the unknown on both sides of the equation<br>• find approximate solutions to linear equations using a graph<br>• compare lengths, areas and volumes using ratio notation<br>• calculate perimeters of 2D shapes, including circles<br>• identify and apply circle definitions and properties, including: centre, radius, chord, diameter, circumference<br>• know the formulae: circumference of a circle = $2\pi r = \pi d$, area of a circle = $\pi r^2$ |
| Calculating | 12 | |
| Visualising and constructing | 12 | |
| Algebraic proficiency: tinkering | 12 | |
| Exploring fractions, decimals and percentages | 4 | |
| Proportional reasoning | 12 | |
| Pattern sniffing | 4 | |
| Investigating angles | 8 | |
| Calculating fractions, decimals and percentages | 8 | |
| Solving equations and inequalities | 4 | |
| Calculating space | 12 | |





| | | |
|---|---|---|
| Algebraic proficiency: visualising | 12 | • calculate exactly with multiples of pi (NEW)<br>• calculate areas of circles and composite shapes<br>• know and apply formulae to calculate volume of right prisms (including cylinders)<br>• calculate the surface area of right prisms (including cylinders) (NEW)<br>• know the formulae for: Pythagoras' theorem, $a^2 + b^2 = c^2$, and apply it to find lengths in right-angled triangles in two dimensional figures (NEW)<br>• identify and interpret gradients and intercepts of linear functions graphically<br>• recognise, sketch and interpret graphs of linear functions and simple quadratic functions<br>• plot and interpret graphs and graphs of non-standard (piece-wise linear) functions in real contexts, to find approximate solutions to problems such as simple kinematic problems involving distance and speed<br>• apply systematic listing strategies<br>• enumerate sets and combinations of sets systematically, using tables, grids and Venn diagrams<br>• construct theoretical possibility spaces for combined experiments with equally likely outcomes and use these to calculate theoretical probabilities<br>• apply the property that's the possibilities of an exhaustive set of outcomes sum to one; applt the property that the probabilities of an exhaustive set of mutually exclusive events sum to one (NEW)<br>• interpret, analyse and compare the distributions of data sets from univariate empirical distributions through appropriate graphical representation involving discrete, continuous and grouped data<br>• use and interpret scatter graphs of bivariate data<br>• draw estimated lines of best fit; make predictions (NEW)<br>• recognise correlation<br>• interpret, analyse and compare the distributions of data sets from univariate empirical distributions through appropriate measures of central tendency (median, mean, mode and modal class) and spread (range, including consideration of outliers)<br>• apply statistics to describe a population |
| Understanding risk II | 8 | |
| Presentation of data | 8 | |
| Measuring data | 8 | |
| | | |

| Numbers and the number system | Link to Stage 9 | 9 hours |
|---|---|---|

**KNOWLEDGE**

**The Big Picture**: Number and Place Value progression map

- **interpret standard form $A \times 10^n$, where $1 \leq A < 10$ and $n$ is an integer**
- **use inequality notation to specify simple error intervals due to truncation or rounding (NEW)**
- **apply and interpret limits of accuracy (NEW)**

## SKILLS

- Rounding to any significant figure
- Expressing error intervals algebraically.
- Using and identifying upper and lower bounds
- Truncating
- Write a large (small) number in standard form
- Interpret a large (small) number written in standard form
- Recall of indices laws (including negative powers)*

*Covered in next key concept – introduction/recap only required

## Suggested resources

KM: Astronomical numbers
KM: Interesting standard form
KM: Powers of ten
KM: Maths to Infinity: Standard form
Powers of ten film (external site)
The scale of the universe animation (external site)

**Learning review**

- www.diagnosticquestions.com

## Prerequisites

- Know the meaning of a prime number
- Recall prime numbers up to 50
- Understand the use of notation for powers
- Know how to round to the nearest whole number, 10, 100, 1000 and to decimal places
- Multiply and divide numbers by powers of 10

## Mathematical language

Prime
Prime factor
Prime factorisation
Product
Venn diagram
Highest common factor
Lowest common multiple
Standard form
Significant figure

**Notation**

Index notation: e.g. $5^3$ is read as '5 to the power of 3' and means '3 lots of 5 multiplied together'
Standard form (see key concepts) is sometimes called 'standard index form', or more properly, 'scientific notation'

## Agreed Common Teaching Approaches

Pupils should explore the ways to enter and interpret numbers in standard form on a scientific calculator. Different calculators may very well have different displays, notations and methods.
Liaise with the science department to establish when students first meet the use of standard form, and in what contexts they will be expected to interpret it.
NRICH: Divisibility testing
NCETM: Glossary

**Common approaches**

*The following definition of a prime number should be used in order to minimise confusion about 1: A prime number is a number with exactly two factors.*
*The description 'standard form' is always used instead of 'scientific notation' or 'standard index form'*

## Reasoning opportunities and probing questions

- Show me two (three-digit) numbers with a highest common factor of 18. And another. And another…
- Show me two numbers with a lowest common multiple of 240. And another. And another…
- Jenny writes $7.1 \times 10^{-5} = 0.0000071$. Kenny writes $7.1 \times 10^{-5} = 0.000071$. Who do you agree with? Why?

## Cross Curricular Links

## Possible misconceptions

- Many pupils believe that 1 is a prime number – a misconception which can arise if the definition is taken as 'a number which is divisible by itself and 1'
- Some pupils may think 35 934 = 36 to two significant figures
- When converting between ordinary and standard form some pupils may incorrectly connect the power to the number of zeros; e.g. $4 \times 10^5 = 400\,000$ so $4.2 \times 10^5 = 4\,200\,000$
- Similarly, when working with small numbers (negative powers of 10) some pupils may think that the power indicates how many zeros should be placed between the decimal point and the first non-zero digit

| Calculating | Link to Stage 9 | 9 hours |
|---|---|---|

**KNOWLEDGE**

**The Big Picture**: Calculation progression map

- **apply the four operations, including formal written methods, to integers, decimals and simple fractions (proper and improper), and mixed numbers – all both positive and negative**
- **use conventional notation for priority of operations, including brackets, powers, roots and reciprocals**
- **calculate with roots, and with integer indices (NEW)**
- **calculate exactly with fractions**

## SKILLS

- Fractions: all operations (including mixed numbers)
- Decimals: al operations (including mixed numbers)
- Adding, multiplying, dividing and multiplying negative numbers
- Know how to square (or cube) a negative number (e.g $-3^2$ or $-4^3$)
- Enter negative numbers into a calculator e.g $(-1)^2 = 1$
- Interpret a calculator display when working with negative numbers
- Understand how to use the order of operations including powers, roots, brackets and division
- Know the value of square and cube roots, including understanding positive and negative solutions.
- Find fractions of amounts

## Suggested Resources

KM: Summing up
KM: Developing negatives
KM: Sorting calculations
KM: Maths to Infinity: Directed numbers
Standards Unit: N9 Evaluating directed number statements
NRICH: Working with directed numbers

**Learning review**

- www.diagnosticquestions.com

## Prerequisites

- Fluently recall and apply multiplication facts up to 12 × 12
- Know and use column addition and subtraction
- Know the formal written method of long multiplication
- Know the formal written method of short division
- Convert between an improper fraction and a mixed number
- Know the order of operations for the four operations and brackets

**Bring on the Maths⁺: Moving on up!**
Number and Place Value: v3

## Mathematical language

Negative number
Directed number
Improper fraction
Top-heavy fraction
Mixed number
Operation
Inverse
Long multiplication
Short division
Power
Indices
Roots

## Agreed Common Teaching Approaches

Pupils need to know how to enter negative numbers into their calculator and how to interpret the display.
The grid method is promoted as a method that aids numerical understanding and later progresses to multiplying algebraic statements.
NRICH: Adding and subtracting positive and negative numbers
NRICH: History of negative numbers
NCETM: Departmental workshop: Operations with Directed Numbers
NCETM: Glossary

**Common approaches**
*Teachers use the language 'negative number', and not 'minus number', to avoid confusion with calculations*
*Every classroom has a negative number washing line on the wall*
*Long multiplication and short division are to be promoted as the 'most efficient methods'.*
*If any acronym is promoted to help remember the order of operations, then BIDMAS is used as the I stands for indices.*

## Reasoning opportunities and probing questions

- Convince me that -3 - -7 = 4
- Show me an example of a calculation involving addition of two negative numbers and the solution -10. And another. And another …
- Create a Carroll diagram with 'addition', 'subtraction' as the column headings and 'one negative number', 'two negative numbers' as the row headings. Ask pupils to create (if possible) a calculation that can be placed in each of the four positions. If they think it is not possible, explain why. Repeat for multiplication and division.

## Cross Curricular Links

## Possible misconceptions

- Some pupils may use a rule stated as 'two minuses make a plus' and make many mistakes as a result; e.g. -4 + -6 = 10
- Some pupils may incorrectly apply the principle of commutativity to subtraction; e.g. 4 – 7 = 3
- The order of operations is often not applied correctly when squaring negative numbers. As a result pupils may think that $x^2 = -9$ when x = -3. The fact that a calculator applies the correct order means that $-3^2 = -9$ and this can actually reinforce the misconception. In this situation brackets should be used as follows: $(-3)^2 = 9$.

| Visualising and constructing | Link to Stage 9 | 9 hours |
|---|---|---|

**KNOWLEDGE**

**The Big Picture**: Properties of Shape progression map

- **measure line segments and angles in geometric figures, including interpreting maps and scale drawings and use of bearings**
- **identify, describe and construct similar shapes, including on coordinate axes, by considering enlargement**
- **interpret plans and elevations of 3D shapes**
- **use scale factors, scale diagrams and maps**

## SKILLS

- Know the vocabulary of enlargement
- Find the centre of enlargement
- Find the scale factor of an enlargement
- Use the centre and scale factor to carry out an enlargement with positive integer (fractional) scale factor
- Know and understand the vocabulary of plans and elevations
- Interpret plans and elevations
- Draw plans and elevations
- Using bearings in real-life context
- Use bearings to solve geometrical problems
- Calculating a bearing
- Drawing a bearing
- Understanding bearings are 3 figures
- Use and interpret scale drawings (map scales, design plans, blueprints etc.)

## Suggested Resources

KM: Outdoor Leisure 13
KM: Airports and hilltops
KM: Plans and elevations
KM: Transformation template
KM: Enlargement I
KM: Enlargement II
KM: Investigating transformations with Autograph (enlargement and Main Event II). Dynamic example.
WisWeb applet: Building houses
NRICH: Who's the fairest of them all?

**Learning review**

- www.diagnosticquestions.com

## Prerequisites

- Use a protractor to measure angles to the nearest degree
- Use a ruler to measure lengths to the nearest millimetre
- Understand coordinates in all four quadrants
- Work out a multiplier given two numbers

## Mathematical language

Similar, Similarity
Enlarge, enlargement
Scaling
Scale factor
Centre of enlargement
Object
Image
Scale drawing
Bearing
Plan, Elevation

**Notation**

Bearings are always given as three figures; e.g. 025°.
Cartesian coordinates: separated by a comma and enclosed by brackets

## Agreed Common Teaching Approaches

Describing enlargement as a 'scaling' will help prevent confusion when dealing with fractional scale factors
NCETM: Departmental workshops: Enlargement
NCETM: Glossary

**Common approaches**

*All pupils should experience using dynamic software (e.g. Autograph) to visualise the effect of moving the centre of enlargement, and the effect of varying the scale factor.*

## Reasoning opportunities and probing questions

- Give an example of a shape and its enlargement (e.g. scale factor 2) with the guidelines drawn on. How many different ways can the scale factor be derived?
- Show me an example of a sketch where the bearing of A from B is between 90° and 180°.
- The bearing of A from B is 'x'. Find the bearing of B from A in terms of 'x'. Explain why this works.
- Provide the plan and elevations of shapes made from some cubes. Challenge pupils to build the shape and place it in the correct orientation.

## Cross Curricular Links

## Possible misconceptions

- Some pupils may think that the centre of enlargement always has to be (0,0), or that the centre of enlargement will be in the centre of the object shape.
- If the bearing of A from B is 'x', then some pupils may think that the bearing of B from A is '180 – x'.
- The north elevation is the view of a shape from the north (the north face of the shape), not the view of the shape while facing north.

| Algebraic proficiency: tinkering | Link to Stage 9 | 9 hours |
|---|---|---|

**KNOWLEDGE**

**The Big Picture**: Algebra progression map

- **use and interpret algebraic notation, including: $a^2b$ in place of $a \times a \times b$, coefficients written as fractions rather than as decimals**
- **understand and use the concepts and vocabulary of factors**
- **simplify and manipulate algebraic expressions by taking out common factors and simplifying expressions involving sums, products and powers, including the laws of indices**
- **substitute numerical values into scientific formulae**

## SKILLS

- Know how to write products algebraically
- Use fractions when working in algebraic situations
- Identify common factors (numerical and algebraic) of terms in an expression
- Factorise an expression by taking out common factors
- Simplify an expression involving terms with combinations of variables (e.g. $3a^2b + 4ab^2 + 2a^2 – a^2b$)
- Expanding single brackets
- Forming and solving equations
- Know the multiplication (division, power, zero) law of indices (algebraic context)
- Substitute positive and negative numbers into scientific formulae
- Be aware of common scientific formulae (see Cross Curricular Links)

## Suggested Resources

KM: Missing powers
KM: Laws of indices. Some useful questions.
KM: Maths to Infinity: Indices
NRICH: Temperature

**Learning review**

- www.diagnosticquestions.com

## Prerequisites

- Know basic algebraic notation (the rules of algebra)
- Simplify an expression by collecting like terms
- Know how to multiply a single term over a bracket
- Substitute positive numbers into expressions and formulae
- Calculate with negative numbers

## Mathematical language

Product
Variable
Term
Coefficient
Common factor
Factorise
Power
Indices
Formula, Formulae
Subject
Change the subject

**Notation**
See key concepts above

## Agreed Common Teaching Approaches

During this unit pupils should experience factorising a quadratic expression such as $6x^2 + 2x$.
Collaborate with the science department to establish a list of formulae that will be used, and ensure consistency of approach and experience.
NCETM: Algebra
NCETM: Departmental workshop: Index Numbers
NCETM: Departmental workshops: Deriving and Rearranging Formulae
NCETM: Glossary

**Common approaches**
*Once the laws of indices have been established, all teachers refer to 'like numbers multiplied, add the indices' and 'like numbers divided, subtract the indices. They also generalise to $a^m \times a^n = a^{m+n}$, etc.*
*When changing the subject of a formula the principle of balancing (doing the same to both sides) must be used rather than a 'change side, change sign' approach.*

## Reasoning opportunities and probing questions

- Establish the multiplication, division and power laws of indices by writing products in full. Use the division law of indices to establish why $a^0 = 1$.
- What is wrong with this statement and how can it be corrected: $5^2 \times 5^4 = 5^8$ ?
- Jenny thinks that if $y = 2x + 1$ then $x = (y – 1)/2$. Kenny thinks that if $y = 2x + 1$ then $x = y/2 – 1$. Who do you agree with? Why?

## Cross Curricular Links

## Possible misconceptions

- Some pupils may misapply the order of operation when changing the subject of a formula
- Many pupils may think that $a^0 = 0$
- Some pupils may not consider 4ab and 3ba as 'like terms' and therefore will not 'collect' them when simplifying expressions

| Exploring fractions, decimals and percentages | Link to Stage 9 | 3 hours |
|---|---|---|

**KNOWLEDGE**

**The Big Picture**: Fractions, decimals and percentages progression map

- **work interchangeably with terminating decimals and their corresponding fractions (such as 3.5 and 7/2 or 0.375 or 3/8)**

| SKILLS | Suggested Resources |
|---|---|
| • Identify if a fraction is terminating or recurring<br>• Recall some decimal and fraction equivalents (e.g. tenths, fifths, eighths)<br>• Converting between fractions, decimals and percentages with a calculator<br>• Converting between fractions, decimals and percentages without a calculator<br>• Simplify fractions and finding equivalent fractions<br>• Convert a fraction to a decimal by scaling (when possible) or division | KM: FDP conversion. Templates for taking notes.<br>KM: Fraction sort. Tasks one and two only.<br>KM: Maths to Infinity: Fractions, decimals, percentages, ratio, proportion<br>NRICH: Matching fractions, decimals and percentages<br><br>**Learning review**<br>• www.diagnosticquestions.com |

| Prerequisites | Mathematical language | Agreed Common Teaching Approaches |
|---|---|---|
| • Understand that fractions, decimals and percentages are different ways of representing the same proportion<br>• Convert between mixed numbers and top-heavy fractions<br>• Write one quantity as a fraction of another | Fraction<br>Mixed number<br>Top-heavy fraction<br>Percentage<br>Decimal<br>Proportion<br>Terminating<br>Recurring<br>Simplify, Cancel<br><br>**Notation**<br>Diagonal and horizontal fraction bar | The diagonal fraction bar (solidus) was first used by Thomas Twining (1718) when recorded quantities of tea. The division symbol (÷) is called an obelus, but there is no name for a horizontal fraction bar.<br>NRICH: History of fractions<br>NRICH: Teaching fractions with understanding<br>NCETM: Glossary<br><br>**Common approaches**<br>*All pupils should use the horizontal fraction bar to avoid confusion when fractions are coefficients in algebraic situations* |

| Reasoning opportunities and probing questions | Cross Curricular Links | Possible misconceptions |
|---|---|---|
| • Without using a calculator, convince me that 3/8 = 0.375<br>• Show me a fraction / decimal / percentage equivalent. And another. And another …<br>• What is the same and what is different: 2.5, 25%, 0.025 ? | | • Some pupils may make incorrect links between fractions and decimals such as thinking that 1/5 = 0.15<br>• Some pupils may think that 5% = 0.5, 4% = 0.4, etc.<br>• Some pupils may think it is not possible to have a percentage greater than 100%. |



| *Proportional reasoning* | Link to Stage 9 | *9 hours* |
|---|---|---|

**KNOWLEDGE**

**The Big Picture**: Ratio and Proportion progression map

- **express the division of a quantity into two parts as a ratio; apply ratio to real contexts and problems (such as those involving conversion, comparison, scaling, mixing, concentrations)**
- **identify and work with fractions in ratio problems**
- **understand and use proportion as equality of ratios**
- **express a multiplicative relationship between two quantities as a ratio or a fraction**
- **use compound units (such as speed, rates of pay, unit pricing)**
- **change freely between compound units (e.g. speed, rates of pay, prices) in numerical contexts**
- **relate ratios to fractions and to linear functions**

## SKILLS

- Solve problems in ratio in a real-life context (e.g. sharing in ratio)
- Simplify ratio
- Sharing ratio – find total of ratio shares
- Sharing ratio – find individual share
- Use ratio as part in a problem solving context (scaling etc)
- Identify proportion in a situation
- Find a relevant multiplier in a situation involving proportion
- Use fractions fluently in situations involving ratio or proportion
- Understand the connections between ratios, fractions and proportion
- Understand the meaning of a compound unit
- Know the connection between speed, distance and time
- Solve problems involving speed, distance and time
- Identify when it is necessary to convert quantities in order to use a sensible unit of measure
- Solve problems involving proportional reasoning
- Solve problems involving compound units
- Be able to write a ratio as a fraction.
- Recognise ratio and scale in the context of a conversion graph

## Suggested Resources

KM: Proportion for real
KM: Investigating proportionality
KM: Maths to Infinity: Fractions, decimals, percentages, ratio, proportion
NRICH: In proportion
Standards Unit: N6 Developing proportional reasoning

**Learning review**

- www.diagnosticquestions.com

## Prerequisites

- Understand and use ratio notation
- Divide an amount in a given ratio

## Mathematical language

Ratio
Proportion
Proportional
Multiplier
Speed
Unitary method
Units
Compound unit

**Notation**

Kilometres per hour is written as km/h or $kmh^{-1}$
Metres per second is written as m/s or $ms^{-1}$

## Agreed Common Teaching Approaches

NRICH: Ratio or proportion?
NRICH: Roasting old chestnuts 3
NCETM: The Bar Model
NCETM: Multiplicative reasoning
NCETM: Departmental workshops: Proportional Reasoning
NCETM: Glossary

**Common approaches**

*All pupils are taught to set up a 'proportion table' and use it to find the multiplier in situations involving proportion*

## Reasoning opportunities and probing questions

- Show me an example of two quantities that will be in proportion. And another. And another …
- (Showing a table of values such as the one below) convince me that this information shows a proportional relationship

| | |
|---|---|
| 6 | 9 |
| 10 | 15 |
| 14 | 21 |

- Which is the faster speed: 60 km/h or 10 m/s? Explain why.

## Cross Curricular Links

## Possible misconceptions

- Many pupils will want to identify an additive relationship between two quantities that are in proportion and apply this to other quantities in order to find missing amounts
- Some pupils may think that a multiplier always has to be greater than 1
- When converting between times and units, some pupils may base their working on 100 minutes = 1 hour

| Pattern sniffing | Link to Stage 9 | 3 hours |
|---|---|---|

**KNOWLEDGE**

**The Big Picture**: Algebra progression map

- **generate terms of a sequence from either a term-to-term or a position-to-term rule**
- **deduce expressions to calculate the nth term of linear sequences (ALSO 7)**

Return to overview

## SKILLS

- Explore sequences
- Generate a sequence from a term-to-term rule
- Understand the meaning of a position-to-term rule
- Use a position-to-term rule to generate a sequence
- Find the position-to-term rule for a given sequence
- Use algebra to describe the position-to-term rule of a linear sequence (the nth term)
- Find the position-to-term rule for a given sequence from a diagram
- Find the nth term rule for a given sequence from a diagram
- Describe an nth term sequences involving decimals and decreasing sequences
- Describe an nth term sequence involving fractions
- Use the nth term of a sequence to deduce if a given number is in a sequence

## Suggested Resources

KM: Spreadsheet sequences
KM: Generating sequences
KM: Maths to Infinity: Sequences
KM: Stick on the Maths: Linear sequences
NRICH: Charlie's delightful machine
NRICH: A little light thinking
NRICH: Go forth and generalise

**Learning review**

- www.diagnosticquestions.com

## Prerequisites

- Use a term-to-term rule to generate a sequence
- Find the term-to-term rule for a sequence
- Describe a sequence using the term-to-term rule

## Mathematical language

## Agreed Common Teaching Approaches

Using the nth term for times tables is a powerful way of finding the nth term for any linear sequence. For example, if the pupils understand the 3 times table can ne described as '3n' then the linear sequence 4, 7, 10, 13, … can be described as the 3 times table 'shifted up' one place, hence 3n + 1.
Exploring statements such as 'is 171 is in the sequence 3, 9, 15, 21, 27, ..?' is a very powerful way for pupils to realise that 'term-to-term' rules can be inefficient and therefore 'position-to-term' rules (nth term) are needed.
NCETM: Algebra
NCETM: Glossary

**Common approaches**
*Teachers refer to a sequence such as 2, 5, 8, 11, … as 'the three times table minus one', to help pupils construct their understanding of the nth term of a sequence.*
*All students have the opportunity to use spreadsheets to generate sequences*

## Reasoning opportunities and probing questions

- Show me a sequence that could be generated using the nth term 4n ± c. And another. And another …
- What's the same, what's different: 4, 7, 10, 13, 16, …. , 2, 5, 8, 11, 14, … , 4, 9, 14, 19, 24, …. and 4, 10, 16, 22, 28, …?
- The 4th term of a linear sequence is 15. Show me the nth term of a sequence with this property. And another. And another …
- Convince me that the nth term of the sequence 2, 5, 8, 11, … is 3n -1 .
- Kenny says the 171 is in the sequence 3, 9, 15, 21, 27, … Do you agree with Kenny? Explain your reasoning.

## Cross Curricular

## Possible misconceptions

- Some pupils will think that the nth term of the sequence 2, 5, 8, 11, … is n + 3.
- Some pupils may think that the (2n)th term is double the nth term of a linear sequence.
- Some pupils may think that sequences with nth term of the form 'ax ± b' must start with 'a'.

| *Investigating angles* | Link to Stage 9 | 6 hours |
|---|---|---|

**KNOWLEDGE**

**The Big Picture**: Position and direction progression map

- **understand and use alternate and corresponding angles on parallel lines**
- **derive and use the sum of angles in a triangle (e.g. to deduce and use the angle sum in any polygon, and to derive properties of regular polygons)**

Return to overview

| SKILLS | Suggested Resources |
|---|---|
| • Explore geometrical situations involving parallel lines<br>• Identify alternate angles and know that they are equal<br>• Identify corresponding angles and know that they are equal<br>• Identify vertically opposite angles amd know they are equal<br>• Use knowledge of alternate and corresponding angles to calculate missing angles in geometrical diagrams<br>• Establish the fact that angles in a triangle must total 180°<br>• Use the fact that angles in a triangle total 180° to work out the total of the angles in any polygon<br>• Establish the size of an interior angle in a regular polygon<br>• Know the total of the exterior angles in any polygon<br>• Establish the size of an exterior angle in a regular polygon | KM: Alternate and corresponding angles<br>KM: Perplexing parallels<br>KM: Investigating polygons<br>KM: Maths to Infinity: Lines and angles<br>KM: Stick on the Maths: Alternate and corresponding angles<br>KM: Stick on the Maths: Geometrical problems<br>NRICH: Ratty<br><br>**Learning review**<br>• www.diagnosticquestions.com |

| Prerequisites | Mathematical language | Pedagogical notes |
|---|---|---|
| • Use angles at a point, angles at a point on a line and vertically opposite angles to calculate missing angles in geometrical diagrams<br>• Know that the angles in a triangle total 180° | Degrees<br>Right angle, acute angle, obtuse angle, reflex angle<br>Vertically opposite<br>Geometry, geometrical<br>Parallel<br>Alternate angles, corresponding angles<br>Interior angle, exterior angle<br>Regular polygon<br><br>**Notation**<br>Dash notation to represent equal lengths in shapes and geometric diagrams<br>Arrow notation to show parallel lines | The KM: Perplexing parallels resource is a great way for pupils to discover practically the facts for alternate and corresponding angles.<br>Pupils have established the fact that angles in a triangle total 180° in Stage 7. However, using alternate angles they are now able to prove this fact.<br>Encourage pupils to draw regular and irregular convex polygons to discover the sum of the interior angles = $(n - 2) \times 180°$.<br>NCETM: Glossary<br><br>**Common approaches**<br>*Teachers insist on correct mathematical language (and not F-angles or Z-angles for example)* |

| Reasoning opportunities and probing questions | Suggested resources | Possible misconceptions |
|---|---|---|
| • Show me a pair of alternate (corresponding) angles. And another. And another …<br>• Jenny thinks that hexagons are the only polygon that tessellates. Do you agree? Explain your reasoning.<br>• Convince me that the angles in a triangle total 180°.<br>• Convince me that the interior angle of a pentagon is 540°.<br>• Always/ Sometimes/ Never: The sum of the interior angles of an n-sided polygon can be calculated using sum = $(n - 2) \times 180°$.<br>• Always/ Sometimes/ Never: The sum of the exterior angles of a polygon is 360°. | | • Some pupils may think that alternate and/or corresponding angles have a total of 180° rather than being equal.<br>• Some pupils may think that the sum of the interior angles of an n-sided polygon can be calculated using Sum = $n \times 180°$.<br>• Some pupils may think that the sum of the exterior angles increases as the number of sides of the polygon increases. |



| Calculating fractions, decimals and percentages | Link to Stage 9 | 6 hours |
|---|---|---|

**KNOWLEDGE**

**The Big Picture**: Fractions, decimals and percentages progression map

- **interpret fractions and percentages as operators**
- **work with percentages greater than 100%**
- **solve problems involving percentage change, including original value problems, and simple interest including in financial mathematics**

Return to overview

## SKILLS

- Calculate with fractions (all operations)
- Recognise when a fraction (percentage) should be interpreted as a number
- Recognise when a fraction (percentage) should be interpreted as a operator
- Identify the multiplier for a percentage increase or decrease when the percentage is greater than 100%
- Use calculators to increase an amount by a percentage greater than 100%
- Find percentage of an amount
- Convert a test score/rating into a percentage
- Solve problems involving percentage change
- Solve original value problems when working with percentages
- Solve financial problems including simple interest
- Introduce the idea of compound interest
- Understand the meaning of giving an exact solution
- Solve problems that require exact calculation with fractions

## Suggested resources

KM: Stick on the Maths: Proportional reasoning
KM: Stick on the Maths: Multiplicative methods
KM: Percentage identifying
NRICH: One or both
NRICH: Antiques roadshow

**Learning review**

- www.diagnosticquestions.com

## Prerequisites

- Apply the four operations to proper fractions, improper fractions and mixed numbers
- Use calculators to find a percentage of an amount using multiplicative methods
- Identify the multiplier for a percentage increase or decrease
- Use calculators to increase (decrease) an amount by a percentage using multiplicative methods
- Know that percentage change = actual change ÷ original amount

## Mathematical language

Proper fraction, improper fraction, mixed number
Simplify, cancel, lowest terms
Percent, percentage
Percentage change
Original amount
Multiplier
(Simple) interest
Exact

**Notation**
Mixed number notation
Horizontal / diagonal bar for fractions

## Agreed Common Approaches

The bar model is a powerful strategy for pupils to 're-present' a problem involving percentage change.
Only simple interest should be explored in this unit. Compound interest will be developed later.
NCETM: The Bar Model
NCETM: Glossary

**Common approaches**
*When adding and subtracting mixed numbers pupils are taught to convert to improper fractions as a general strategy*
*Teachers use the horizontal fraction bar notation at all times*

## Reasoning opportunities and probing questions

- Convince me that the multiplier for a 150% increase is 2.5
- Kenny buys a poncho in a 25% sale. The sale price is £40. Kenny thinks that the original is £50. Do you agree with Kenny? Explain your answer.
- Jenny thinks that increasing an amount by 200% is the same as multiplying by 3. Do you agree with Jenny? Explain your answer.

## Cross Curricular Links

## Possible misconceptions

- Some pupils may think that the multiplier for a 150% increase is 1.5
- Some pupils may think that increasing an amount by 200% is the same as doubling.
- In isolation, pupils may be able to solve original value problems confidently. However, when it is necessary to identify the type of percentage problem, many pupils will apply a method for a more simple percentage increase / decrease problem. If pupils use models (e.g. the bar model, or proportion tables) to represent all problems then they are less likely to make errors in identifying the type of problem.

| Solving equations and inequalities | Link to Stage 9 | 3 hours |
|---|---|---|

**KNOWLEDGE**

**The Big Picture**: Algebra progression map

- **solve linear equations with the unknown on both sides of the equation**
- **find approximate solutions to linear equations using a graph**

Return to overview

## SKILLS

- Solve linear equations with the unknown on one side
- Solve linear equations with the unknown on both sides
- Explore connections between graphs and equations
- Identify the correct order of undoing the operations in an equation
- Solve linear equations with the unknown on one side when the solution is a negative number
- Solve linear equations with the unknown on both sides when the solution is a whole number
- Solve linear equations with the unknown on both sides when the solution is a fraction
- Solve linear equations with the unknown on both sides when the solution is a negative number
- Solve linear equations with the unknown on both sides when the equation involves brackets
- Recognise that the point of intersection of two graphs corresponds to the solution of a connected equation
- Check the solution to an equation by substitution

## Suggested resources

KM: Solving equations
KM: Stick on the Maths: Constructing and solving equations
NRICH: Think of Two Numbers

**Learning review**

- www.diagnosticquestions.com

## Prerequisites

- Choose the required inverse operation when solving an equation
- Solve linear equations by balancing when the solution is a whole number or a fraction

## Mathematical language

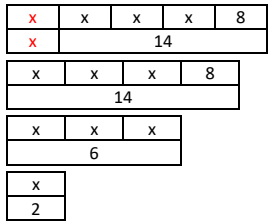
Algebra, algebraic, algebraically
Unknown
Equation
Operation
Solve
Solution
Brackets
Symbol
Substitute
Graph
Point of intersection

**Notation**

The lower case and upper case of a letter should not be used interchangeably when worked with algebra
Juxtaposition is used in place of '×'. 2a is used rather than a2.
Division is written as a fraction

## Agreed Common Teaching Approaches

This unit builds on the wok solving linear equations with unknowns on one side in Stage 7. It is essential that pupils are secure with solving these equations before moving onto unknowns on both sides.
Encourage pupils to 're-present' the problem using the Bar Model.
NCETM: The Bar Model
NCETM: Algebra
NCETM: Glossary




**Common approaches**

*All pupils should solve equations by balancing:*

$4x + 8 = 14 + x$
$-x \quad -x$
$3x + 8 = 14$
$-8 \quad -8$
$3x = 6$
$\div 3 \quad \div 3$
$x = 2$

## Reasoning opportunities and probing questions

- Show me an (one-step, two-step) equation with a solution of -8 (negative, fractional solution). And another. And another …
- Show me a two-step equation that is 'easy' to solve. And another. And another …
- What's the same, what's different: 2x + 7 = 25, 3x + 7 = x + 25, x + 7 = 7 – x, 4x + 14 = 50 ?
- Convince me how you could use graphs to find solutions, or estimates, for equations.

## Cross Curricular Links

## Possible misconceptions

- Some pupils may think that you always have to manipulate the equation to have the unknowns on the LHS of the equal sign, for example 2x – 3 = 6x + 6
- Some pupils think if 4x = 2 then x = 2.
- When solving equations of the form 2x – 8 = 4 – x, some pupils may subtract 'x' from both sides.

| Calculating space | Link to Stage 9 | 9 hours |
|---|---|---|

**KNOWLEDGE**

**The Big Picture**: Measurement and mensuration progression map

- **compare lengths, areas and volumes using ratio notation**
- **calculate perimeters of 2D shapes, including circles**
- **identify and apply circle definitions and properties, including: centre, radius, chord, diameter, circumference**
- **know the formulae: circumference of a circle = $2\pi r = \pi d$, area of a circle = $\pi r^2$**
- **calculate exactly with multiples of pi (NEW)**
- **calculate areas of circles and composite shapes**
- **know and apply formulae to calculate volume of right prisms (including cylinders)**
- **calculate the surface area of right prisms (including cylinders) (NEW)**
- **know the formulae for: Pythagoras' theorem, $a^2 + b^2 = c^2$, and apply it to find lengths in right-angled triangles in two dimensional figures (NEW)**

Return to overview

## SKILLS

- Know the vocabulary of circles
- Know that the number π (pi) = 3.1415926535…
- Recall π to two decimal places
- Know the formula circumference of a circle = $2\pi r = \pi d$
- Calculate the circumference of a circle when radius (diameter) is given
- Calculate the radius (diameter) of a circle when the circumference is known
- Calculate the perimeter of composite shapes that include sections of a circle
- Know the formula area of a circle = $\pi r^2$
- Calculate the area of a circle when radius (diameter) is given
- Calculate the radius (diameter) of a circle when the area is known
- Calculate the area of composite shapes that include sections of a circle
- Know the formula for finding the volume of a right prism (cylinder)
- Calculate the volume of a right prism (cylinder)
- Investigate circles
- Discover pi
- Explore prisms and cylinders
- Solve problems involving circles
- Use Pythagoras' theorem to find shorter sides and hypotenuse
-

## Suggested Resources

KM: Circle connections
KM: Circle problems
KM: Maths to Infinity: Area and Volume
KM: Stick on the Maths: Circumference and area of a circle
KM: Stick on the Maths: Right prisms
NRICH: Blue and White
NRICH: Efficient Cutting
NRICH: Cola Can

**Learning review**
www.diagnosticquestions.com

## Prerequisites

- Know how to use formulae to find the area of rectangles, parallelograms, triangles and trapezia
- Know how to find the area of compound shapes

## Mathematical language

Circle
Centre
Radius, diameter, chord, circumference
Pi
(Right) prism
Cross-section
Cylinder
Polygon, polygonal
Solid

**Notation**
π
Abbreviations of units in the metric system: km, m, cm, mm, $mm^2$, $cm^2$, $m^2$, $km^2$, $mm^3$, $cm^3$, $km^3$

## Agreed Common Teaching Approches

C = πd can be established by investigating the ratio of the circumference to the diameter of circular objects (wheel, clock, tins, glue sticks, etc.)
Pupils need to understand this formula in order to derive $A = \pi r^2$.
A prism is a solid with constant polygonal cross-section. A right prism is a prism with a cross-section that is perpendicular to the 'length'.
NCETM: Glossary

**Common approaches**
*The area of a circle is derived by cutting a circle into many identical sectors and approximating a parallelogram*
*Every classroom has a set of area posters on the wall*
*The formula for the volume of a prism is 'area of cross-section × length' even if the orientation of the solid suggests that height is required*
*Pupils use area of a trapezium = $\frac{(a+b)h}{2}$ and area of a triangle = area = $\frac{bh}{2}$*

## Reasoning opportunities and probing questions

## Cross Curricular Links

## Possible misconceptions

| | | |
|---|---|---|
| • Convince me C = 2πr = πd.<br>• What is wrong with this statement? How can you correct it? The area of a circle with radius 7 cm is approximately 441 $cm^2$ because $(3 \times 7)^2 = 441$.<br>• Convince me the area of a semi-circle = $\frac{\pi d^2}{4}$<br>• Name a right prism. And another. And another …<br>• Convince me that a cylinder is not a prism | | • Some pupils will work out $(\pi \times \text{radius})^2$ when finding the area of a circle<br>• Some pupils may use the sloping height when finding cross-sectional areas that are parallelograms, triangles or trapezia<br>• Some pupils may think that the area of a triangle = base × height<br>• Some pupils may think that you multiply all the numbers to find the volume of a prism<br>• Some pupils may confuse the concepts of surface area and volume |

| Algebraic proficiency: visualising | Link to Stage 9 | 9 hours |
|---|---|---|

**KNOWLEDGE**

**The Big Picture**: Algebra progression map

- **identify and interpret gradients and intercepts of linear functions graphically**
- **recognise, sketch and interpret graphs of linear functions and simple quadratic functions**
- **plot and interpret graphs and graphs of non-standard (*piece-wise linear)* functions in real contexts, to find approximate solutions to problems such as simple kinematic problems involving distance and speed**

Return to overview

## SKILLS

- Plot and interpret linear graphs
- Plot and interpret quadratic graphs
- Model real situations using linear graphs
- Know that graphs of functions of the form y = mx + c, x ± y = c and ax ± by = c are linear
- Plot graphs of functions of the form y = mx + c (x ± y = c, ax ± by = c)
- Understand the concept of the gradient of a straight line
- Find the gradient of a straight line on a unit grid
- Find the y-intercept of a straight line
- Interpret the meaning of y-intercept and gradient on a straight line (give context)
- Sketch a linear graph
- Distinguish between a linear and quadratic graph
- Plot graphs of quadratic functions of the form $y = x^2 \pm c$
- Sketch a simple quadratic graph
- Plot and interpret graphs of linear functions in real contexts
- Plot and interpret distance-time graphs (speed-time graphs)
- Find approximate solutions to kinematic problems involving distance and speed

## Suggested Resources

KM: Matching graphs
KM: Autograph 1
KM: Autograph 2
KM: The hare and the tortoise

**Learning review**

- KM: 8M11 BAM Task

## Prerequisites

- Use coordinates in all four quadrants
- Write the equation of a line parallel to the x-axis or the y-axis
- Draw a line parallel to the x-axis or the y-axis given its equation
- Identify the lines y = x and y = -x
- Draw the lines y = x and y = -x
- Substitute positive and negative numbers into formulae

## Mathematical language

Plot
Equation (of a graph)
Function
Formula
Linear
Coordinate plane
Gradient
y-intercept
Substitute
Quadratic
Piece-wise linear
Model
Kinematic, Speed, Distance

**Notation**

*y = mx + c*

## Agreed Common Teaching Approaches

When plotting graphs of functions of the form y = mx + c a table of values can be useful. Note that negative number inputs can cause difficulties. Pupils should be aware that the values they have found for linear functions should correspond to a straight line.
NCETM: Glossary

**Common approaches**

*Pupils are taught to use positive numbers wherever possible to reduce potential difficulties with substitution of negative numbers*
*Students plot points with a 'x' and not '•'*
*Students draw graphs in pencil*
*All pupils use dynamic geometry software to explore graphs of functions*

## Reasoning opportunities and probing questions

- Draw a distance-time graph of your journey to school. Explain the key features.
- Show me a point on this line (e.g. y = 2x + 1). And another, and another …
- (Given an appropriate distance-time graph) convince me that Kenny is stationary between 10: 00 a.m. and 10:45 a.m.

## Suggested resources

## Possible misconceptions

- When plotting linear graphs some pupils may draw a line segment that stops at the two most extreme points plotted
- Some pupils may think that a sketch is a very rough drawing. It should still identify key features, and look neat, but will not be drawn to scale
- Some pupils may think that a positive gradient on a distance-time graph corresponds to a section of the journey that is uphill
- Some pupils may think that the graph $y = x^2 + c$ is the graph of $y = x^2$ translated horizontally.

| Understanding risk II | Link to Stage 9 | 6 hours |
|---|---|---|

**KNOWLEDGE**

**The Big Picture**: Probability progression map

- **apply systematic listing strategies**
- **enumerate sets and combinations of sets systematically, using tables, grids and Venn diagrams**
- **construct theoretical possibility spaces for combined experiments with equally likely outcomes and use these to calculate theoretical probabilities**
- **apply the property that's the possibilities of an exhaustive set of outcomes sum to one; applt the property that the probabilities of an exhaustive set of mutually exclusive events sum to one (NEW)**

Return to overview

## SKILLS

- Explore experiments and outcomes
- Develop understanding of probability
- Use probability to make predictions
- List all elements in a combination of sets using a Venn diagram
- List outcomes of an event systematically
- Use a table to list all outcomes of an event
- List outcomes of an event using a grid (two-way table)
- Use frequency trees to record outcomes of probability experiments
- Make conclusions about probabilities based on frequency trees
- Understand tree diagrams, and use to solve (AND) type problems.
- Construct theoretical possibility spaces for combined experiments with equally likely outcomes
- Calculate probabilities using a possibility space
- Use theoretical probability to calculate expected outcomes
- Use experimental probability to calculate expected outcomes

## Suggested Resources

KM: Sample spaces
KM: Race game

**Learning review**

- www.diagnosticquestions.com

## Prerequisites

- Convert between fractions, decimals and percentages
- Understand the use of the 0-1 scale to measure probability
- Work out theoretical probabilities for events with equally likely outcomes
- Know how to represent a probability
- Know that the sum of probabilities for all outcomes is 1

## Mathematical language

Outcome
Event
Experiment, Combined experiment
Frequency tree
Enumerate
Set
Venn diagram
Possibility space, sample space
Equally likely outcomes
Theoretical probability
Random
Bias, Fairness
Relative frequency

**Notation**
P(A) for the probability of event A
Probabilities are expressed as fractions, decimals or percentage. They should not be expressed as ratios (which represent odds) or as words

## Agreed Common Approaches

The Venn diagram was invented by John Venn (1834 – 1923)
NCETM: Glossary

**Common approaches**
*All students are taught to use 'DIME' probability recording charts*
*All classes carry out the 'race game' as a simulated horse race with horses numbered 1 to 12*

## Reasoning opportunities and probing questions

- Show me a way of listing all outcomes when two coins are flipped
- Convince me that there are more than 12 outcomes when two six-sided dice are rolled
- Convince me that 7 is the most likely total when two dice are rolled

## Cross Curricular Links

## Possible misconceptions

- Some students may think that there are only three outcomes when two coins are flipped, or that there are only six outcomes when three coins are flipped
- Some students may think that there are 12 unique outcomes when two dice are rolled
- Some students may think that there are 12 possible totals when two dice are rolled

| Presentation of data | Link to Stage 9 | 6 hours |
|---|---|---|

**KNOWLEDGE**

**The Big Picture**: Statistics progression map

- **interpret, analyse and compare the distributions of data sets from univariate empirical distributions through appropriate graphical representation involving discrete, continuous and grouped data**
- **use and interpret scatter graphs of bivariate data**
- **draw estimated lines of best fit; make predictions (NEW)**
- **recognise correlation**

Return to overview

## SKILLS

- Explore types of data.
- Select and interpret appropriate graphs and charts, e.g frequency graphs, polygons, pie charts, compound bar charts
- Construct and interpret graphs
- Know the meaning of continuous data
- Interpret a grouped frequency table for continuous data
- Construct a grouped frequency table for continuous data
- Construct histograms for grouped data with equal class intervals
- Interpret histograms for grouped data with equal class intervals
- Construct and use the horizontal axis of a histogram correctly
- Plot a scatter diagram of bivariate data
- Understand the meaning of 'correlation'
- Interpret a scatter diagram using understanding of correlation

## Suggested Resources

KM: Make a 'human' scatter graph by asking pupils to stand at different points on a giant set of axes.
KM: Gathering data
KM: Spreadsheet statistics
KM: Stick on the Maths HD2: Selecting and constructing graphs and charts
KM: Stick on the Maths HD3: Working with grouped data

**Learning review**

- www.diagnosticquestions.com

## Prerequisites

- Know the meaning of discrete data
- Interpret and construct frequency tables
- Construct and interpret pictograms, bar charts, pie charts, tables and vertical line charts

## Mathematical language

Data
Categorical data, Discrete data
Continuous data, Grouped data
Table, Frequency table
Frequency
Histogram
Scale, Graph
Axis, axes
Scatter graph (scatter diagram, scattergram, scatter plot)
Bivariate data
(Linear) Correlation
Positive correlation, Negative correlation

**Notation**
Correct use of inequality symbols when labeling groups in a frequency table

## Agreed Common Approaches

The word histogram is often misused and an internet search of the word will usually reveal a majority of non-histograms. The correct definition is 'a diagram made of rectangles whose areas are proportional to the frequency of the group'. If the class widths are equal, as they are in this unit of work, then the vertical axis shows the frequency. It is only later that pupils need to be introduced to unequal class widths and frequency density.
Lines of best fit on scatter diagrams are not introduced until Stage 9, although pupils may well have encountered both lines and curves of best fit in science by this time.
NCETM: Glossary

**Common approaches**
*All students collect data about their class's height and armspan when first constructing a scatter diagram*

## Reasoning opportunities and probing questions

- Show me a scatter graph with positive (negative, no) correlation. And another. And another.
- Show me a histogram. And another. And another.
- Kenny thinks that histogram is just a 'fancy' name for a bar chart. Do you agree with Kenny? Explain your answer.
- What's the same and what's different: histogram, scatter diagram, bar chart, pie chart?
- Always/Sometimes/Never: A scatter graph

## Cross Curricular Links

## Possible misconceptions

- Some pupils may label the bar of a histogram rather than the boundaries of the bars
- Some pupils may leave gaps between the bars in a histogram
- Some pupils may misuse the inequality symbols when working with a grouped frequency table

| Measuring data | Link to Stage 9 | 6 hours |
|---|---|---|

**KNOWLEDGE** **The Big Picture**: Statistics progression map

- **interpret, analyse and compare the distributions of data sets from univariate empirical distributions through appropriate measures of central tendency (median, mean, mode and modal class) and spread (range, including consideration of outliers)**
- **apply statistics to describe a population**

Return to overview

## SKILLS

- Investigate averages
- Explore ways of summarising data
- Analyse and compare sets of data
- Find the modal class of set of grouped data
- Find the class containing the median of a set of data
- Find the midpoint of a class
- Calculate an estimate of the mean from a grouped frequency table
- Estimate the range from a grouped frequency table
- Analyse and compare sets of data
- Appreciate the limitations of different statistics (mean, median, mode, range)
- Choose appropriate statistics to describe a set of data
- Justify choice of statistics to describe a set of data

## Suggested Resources

KM: Swillions
KM: Lottery project
NRICH: Half a Minute

**Learning review**

- www.diagnosticquestions.com

## Prerequisites

- Understand the mean, mode and median as measures of typicality (or location)
- Find the mean, median, mode and range of a set of data
- Find the mean, median, mode and range from a frequency table

## Mathematical language

Average
Spread
Consistency
Mean
Median
Mode
Range
Statistic
Statistics
Approximate, Round
Calculate an estimate
Grouped frequency
Midpoint

**Notation**
Correct use of inequality symbols when labeling groups in a frequency table

## Agreed Common Teaching Approaches

The word 'average' is often used synonymously with the mean, but it is only one type of average. In fact, there are several different types of mean (the one in this unit properly being named as the 'arithmetic mean').
NCETM: Glossary

**Common approaches**
*Every classroom has a set of statistics posters on the wall*
*All students are taught to use mathematical presentation correctly when calculating and rounding solutions, e.g. (21 + 56 + 35 + 12) ÷ 30 = 124 ÷ 30 = 41.3 to 1 d.p.*

## Reasoning opportunities and probing questions

- Show me an example of an outlier. And another. And another.
- Convince me why the mean from a grouped set of data is only an estimate.
- What's the same and what's different: mean, modal class, median, range?
- Always/Sometimes/Never: A set of grouped data will have one modal class
- Convince me how to estimate the range for grouped data.

## Cross Curricular Links

## Possible misconceptions

- Some pupils may incorrectly estimate the mean by dividing the total by the numbers of groups rather than the total frequency.
- Some pupils may incorrectly think that there can only be one model class.
- Some pupils may incorrectly estimate the range of grouped data by subtracting the upper bound of the first group from the lower bound of the last group.