

Mathematics overview: Stage 9

Unit	Hours	KNOWLEDGE
Calculating	16	<ul style="list-style-type: none"> calculate with standard form $A \times 10^n$, where $1 \leq A < 10$ and n is an integer apply and interpret limits of accuracy, including upper and lower bounds (NEW) calculate with roots, and with fractional indices (NEW) calculate exactly with surds (NEW) change recurring decimals into their corresponding fractions and vice versa (ALSO 8) use the standard ruler and compass constructions (perpendicular bisector of a line segment, constructing a perpendicular line to a given line from/at a given point, bisecting a given angle) (NEW) use these to construct given figures and solve loci problems: know that the perpendicular distance from a point to a line is the shortest distance to the line (NEW) construct plans and elevations of 3D shapes (NEW) identify, describe and construct similar shapes, including on coordinate axes, by considering enlargement (including fractional scale factors) Refresh understanding of translation, rotation and reflection (NOT ASSESSED) understand and use the concepts and vocabulary of identities know the difference between an equation and an identity simplify and manipulate algebraic expressions by expanding products of two binomials and factorising quadratic expressions of the form $x^2 + bx + c$ solve quadratic equations algebraically by factorising find approximate solutions to quadratic equations using a graph argue mathematically to show algebraic expressions are equivalent, and use algebra to support and construct arguments translate simple situations or procedures into algebraic expressions or formulae solve problems involving direct and inverse proportion including graphical and algebraic representations apply the concepts of congruence and similarity, including the relationships between lengths in similar figures change freely between compound units (e.g. density, pressure) in numerical and algebraic contexts use compound units such as density and pressure NEW STYLE RATIO QUESTIONS (NEW) recognise and use Fibonacci type sequences, quadratic sequences understand and use the concepts and vocabulary of inequalities solve linear inequalities in one variable represent the solution set to an inequality on a number line identify and apply circle definitions and properties, including: tangent, arc, sector and segment calculate arc lengths, angles and areas of sectors of circles know the formulae for: Pythagoras' theorem, $a^2 + b^2 = c^2$, and apply it to find lengths in right-angled triangles in two dimensional figures (ALSO Stage 8) apply and prove the standard circle theorems concerning angles, radii, tangents and chords, and use them to prove related results (NEW) know the trigonometric ratios, $\sin\theta = \text{opposite/hypotenuse}$, $\cos\theta = \text{adjacent/hypotenuse}$, $\tan\theta = \text{opposite/adjacent}$ (NEW) apply it to find angles and lengths in right-angled triangles in two dimensional figures (NEW) use the basic congruence criteria for triangles (SSS, SAS, ASA, RHS) apply angle facts, triangle congruence, similarity and properties of quadrilaterals to conjecture and derive results about angles and sides, including Pythagoras' Theorem and the fact that the base angles of an isosceles triangle are equal, and use known results to obtain simple proofs
Visualising and constructing	12	
Algebraic proficiency: tinkering	12	
Proportional reasoning	12	
Pattern sniffing	8	
Solving equations and inequalities I	8	
Calculating space	16	
Conjecturing	8	
Algebraic proficiency: visualising	16	
Solving equations and inequalities II	12	
Understanding risk	8	



Presentation of data

8

- identify and interpret gradients and intercepts of linear functions algebraically
- use the form $y = mx + c$ to identify parallel lines
- find the equation of the line through two given points, or through one point with a given gradient
- recognise, sketch and interpret graphs of quadratic functions
- recognise, sketch and interpret graphs of simple cubic functions and the reciprocal function $y = 1/x$ with $x \neq 0$
- plot and interpret graphs (including reciprocal graphs) and graphs of non-standard functions in real contexts, to find approximate solutions to problems such as simple kinematic problems involving distance, speed and acceleration
- solve, in simple cases, two linear simultaneous equations in two variables algebraically
- derive an equation (or two simultaneous equations), solve the equation(s) and interpret the solution
- find approximate solutions to simultaneous equations using a graph
- calculate the probability of independent and dependent combined events, including using tree diagrams and other representations, and know the underlying assumptions
- enumerate sets and combinations of sets systematically, using tree diagrams
- understand that empirical unbiased samples tend towards theoretical probability distributions, with increasing sample size
- interpret and construct tables, charts and diagrams, including tables and line graphs for time series data and know their appropriate use
- know correlation does not indicate causation; interpolate and extrapolate apparent trends whilst knowing the dangers of so doing
- construct and interpret diagrams for grouped discrete data and continuous data, i.e. cumulative frequency graphs, and know their appropriate use (NEW)



KNOWLEDGE

The Big Picture: [Calculation progression map](#)

- calculate with standard form $A \times 10^n$, where $1 \leq A < 10$ and n is an integer
- **apply and interpret limits of accuracy, including upper and lower bounds (NEW)**
- **calculate with roots, and with fractional indices (NEW)**
- **calculate exactly with surds (NEW)**
- **change recurring decimals into their corresponding fractions and vice versa (ALSO 8)**

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SKILLS

Suggested Resources

- Calculate with positive indices (roots) using written methods
- Calculate with negative indices in the context of standard form
- Use a calculator to evaluate numerical expressions involving powers (roots)
- Interpret a number written in standard form
- Add (subtract) numbers written in standard form
- Multiply (divide) numbers written in standard form
- Convert a 'near miss' into standard form; e.g. 23×10^7
- Enter a calculation written in standard form into a scientific calculator
- Interpret the standard form display of a scientific calculator
- Understand the difference between truncating and rounding
- Interpret and understand fractional powers
- Identify the minimum and maximum values of an amount that has been rounded (to nearest x , x d.p., x s.f.)
- Find upper and lower bounds
- Use upper and lower bounds in problem solving context
- Express upper and lower bounds using inequalities
- Use inequalities to describe the range of values for a rounded value
- Solve problems involving the maximum and minimum values of an amount that has been rounded
- Understand the definition of a surd
- Multiply and Divide surds
- Expand double brackets involving surds (expanding only)
- Rationalise the denominator of surds (one term)
- Add and subtract surds using simplification.
- Link simplification of surds to prime factorisation.
- Change recurring decimals into fractions using an algebraic method
- Change fractions into recurring decimals using division

KM: [Maths to Infinity: Standard form](#)
 KM: [Maths to Infinity: Indices](#)
 Investigate 'Narcissistic Numbers'.
 NRICH: [Power mad!](#)
 NRICH: [A question of scale](#)
[The scale of the universe](#) animation (external site)

Learning review

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Prerequisites

Mathematical language

Agreed Common Teaching Approaches



<ul style="list-style-type: none"> • Know the meaning of powers • Know the meaning of roots • Know the multiplication and division laws of indices • Understand and use standard form to write numbers • Round to a given number of decimal places or significant figures • Know the meaning of the symbols $<$, $>$, \leq, \geq 	Power Root Index, Indices Standard form Inequality Truncate Round Minimum, Maximum Interval Decimal place Significant figure Notation Standard form: $A \times 10^n$, where $1 \leq A < 10$ and n is an integer Inequalities: e.g. $x > 3$, $-2 < x \leq 5$	Liaise with the science department to establish when students first meet the use of standard form, and in what contexts they will be expected to interpret it. NCETM: Departmental workshops: Index Numbers NCETM: Glossary Common approaches <i>The description 'standard form' is always used instead of 'scientific notation' or 'standard index form'.</i> <i>Standard form is used to introduce the concept of calculating with negative indices. The link between 10^n and $1/10^n$ can be established. The language 'negative number' is used instead of 'minus number'.</i>
Reasoning opportunities and probing questions	Cross curricular links	Possible misconceptions
<ul style="list-style-type: none"> • Kenny thinks this number is written in standard form: 23×10^7. Do you agree with Kenny? Explain your answer. • When a number 'x' is rounded to 2 significant figures the result is 70. Jenny writes '$65 < x < 75$'. What is wrong with Jenny's statement? How would you correct it? • Convince me that $4.5 \times 10^7 \times 3 \times 10^5 = 1.35 \times 10^{13}$ 		<ul style="list-style-type: none"> • Some students may think that any number multiplied by a power of ten qualifies as a number written in standard form • When rounding to significant figures some students may think, for example, that 6729 rounded to one significant figure is 7 • Some students may struggle to understand why the maximum value of a rounded number is actually a value which would not round to that number; i.e. if given the fact that a number 'x' is rounded to 2 significant figures the result is 70, they might write '$65 < x < 74.99$'



KNOWLEDGE

The Big Picture: [Properties of Shape progression map](#)

- use the standard ruler and compass constructions (perpendicular bisector of a line segment, constructing a perpendicular line to a given line from/at a given point, bisecting a given angle) (NEW)
- use these to construct given figures and solve loci problems: know that the perpendicular distance from a point to a line is the shortest distance to the line (NEW)
- construct plans and elevations of 3D shapes (NEW)
- identify, describe and construct similar shapes, including on coordinate axes, by considering enlargement (including fractional scale factors)
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SKILLS

Suggested Resources

- Know the vocabulary of enlargement
- Find the centre of enlargement
- Find the scale factor of an enlargement
- Use the centre and scale factor to carry out an enlargement with positive integer (fractional) scale factor
- Know and understand the vocabulary of plans and elevations
- Interpret plans and elevations
- Use the concept of scaling in diagrams
- Construct triangles given angles and side lengths
- Construct an equilateral triangle
- Construct the perpendicular bisector of a line
- Bisect an angle
- Construct a perpendicular line from a given point
- Create a loci of points equidistant from a point
- Apply construction skills in real-life contexts to solve loci problems.
- Translate a shape using vector notation
- Rotate a shape given angle, direction and centre
- Reflect a shape given line of reflection
- Refresh understanding of translation, rotation and reflection (NOT ASSESSED)

KM: [Outdoor Leisure 13](#)
 KM: [Airports and hilltops](#)
 KM: [Plans and elevations](#)
 KM: [Transformation template](#)
 KM: [Enlargement I](#)
 KM: [Enlargement II](#)
 KM: [Investigating transformations](#) with Autograph (enlargement and Main Event II). [Dynamic example](#).
 WisWeb applet: [Building houses](#)
 NRICH: [Who's the fairest of them all?](#)

Learning review
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Prerequisites

Mathematical language

Agreed Common Teaching Approaches

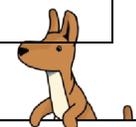
- Use a protractor to measure angles to the nearest degree
- Use a ruler to measure lengths to the nearest millimetre
- Understand coordinates in all four quadrants
- Work out a multiplier given two numbers

Similar, Similarity
 Enlarge, enlargement
 Scaling
 Scale factor
 Centre of enlargement
 Object
 Image
 Scale drawing
 Bearing
 Plan, Elevation

Notation
 Bearings are always given as three figures; e.g. 025°.
 Cartesian coordinates: separated by a comma and enclosed by brackets

Describing enlargement as a 'scaling' will help prevent confusion when dealing with fractional scale factors
 NCETM: [Departmental workshops: Enlargement](#)
 NCETM: [Glossary](#)

Common approaches
All pupils should experience using dynamic software (e.g. Autograph) to visualise the effect of moving the centre of enlargement, and the effect of varying the scale factor.



Reasoning opportunities and probing questions	Cross Curricular Links	Possible misconceptions
<ul style="list-style-type: none"> • Give an example of a shape and its enlargement (e.g. scale factor 2) with the guidelines drawn on. How many different ways can the scale factor be derived? • Show me an example of a sketch where the bearing of A from B is between 90° and 180°. • The bearing of A from B is 'x'. Find the bearing of B from A in terms of 'x'. Explain why this works. • Provide the plan and elevations of shapes made from some cubes. Challenge pupils to build the shape and place it in the correct orientation. 		<ul style="list-style-type: none"> • Some pupils may think that the centre of enlargement always has to be (0,0), or that the centre of enlargement will be in the centre of the object shape. • If the bearing of A from B is 'x', then some pupils may think that the bearing of B from A is '$180 - x$'. • The north elevation is the view of a shape from the north (the north face of the shape), not the view of the shape while facing north.



KNOWLEDGE

The Big Picture: [Algebra progression map](#)

- understand and use the concepts and vocabulary of **identities**
- **know the difference between an equation and an identity**
- **simplify and manipulate algebraic expressions by expanding products of two binomials and factorising quadratic expressions of the form $x^2 + bx + c$**
- **solve quadratic equations algebraically by factorising**
- **find approximate solutions to quadratic equations using a graph**
- **argue mathematically to show algebraic expressions are equivalent, and use algebra to support and construct arguments**
- **translate simple situations or procedures into algebraic expressions or formulae**

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SKILLS

Suggested resources

- Understand the meaning of an identity
- Understand the meanings of expression, equation & formula
- Expanding brackets
- Multiply two linear expressions of the form $(x + a)(x + b)$
- Expand the expression $(x \pm a)^2$
- Simplify an expression involving ' x^2 ' by collecting like terms
- Factorise an expression involving one bracket
- Identify when it is necessary to remove factors to factorise a quadratic expression
- Identify when it is necessary to find two linear expressions to factorise a quadratic expression
- Factorise a quadratic expression of the form $x^2 + bx + c$
- Solve a quadratic expression of the form $x^2 + bx + c$
- Work out why two algebraic expressions are equivalent
- Create a mathematical argument to show that two algebraic expressions are equivalent (forming identity)
- Create an expression or a formula to describe a situation (forming an equation)

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Prerequisites

- Manipulate expressions by collecting like terms
- Know that $x \times x = x^2$
- Calculate with negative numbers
- Know the grid method for multiplying two two-digit numbers
- Know the difference between an expression, an equation and a formula

Mathematical language

Inequality
Identity
Equivalent
Equation
Formula, Formulae
Expression
Expand
Linear
Quadratic

Notation

The equals symbol '=' and the equivalency symbol '≡'

Agreed Common Teaching Approaches

Pupils should be taught to use the equivalency symbol '≡' when working with identities.
During this unit pupils could construct (and solve) equations in addition to expressions and formulae.
See former coursework task, opposite corners
NCETM: [Algebra](#)
NCETM: [Departmental workshops: Deriving and Rearranging Formulae](#)
NCETM: [Glossary](#)

Common approaches

All students are taught to use the grid method to multiply two linear expressions. They then use the same approach in reverse to factorise a quadratic.

Reasoning opportunities and probing questions

- The answer is $x^2 + 10x + c$. Show me a possible question. And another. And another ... (Factorising a quadratic expression of the form $x^2 + bx + c$ can be introduced as a reasoning activity: once pupils are fluent at multiplying two linear expressions they can be asked 'if this is the answer, what is the question?')
- Convince me that $(x + 3)(x + 4)$ does not equal $x^2 + 7$.
- What is wrong with this statement? How can you correct it? $(x + 3)(x + 4) \equiv x^2 + 12x + 7$.
- Jenny thinks that $(x - 2)^2 = x^2 - 4$. Do you agree with Jenny? Explain your answer.

Cross Curricular Links

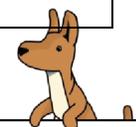
KM: [Stick on the Maths: Multiplying linear expressions](#)
KM: [Maths to Infinity: Brackets](#)
KM: [Maths to Infinity: Quadratics](#)
NRICH: [Pair Products](#)
NRICH: [Multiplication Square](#)
NRICH: [Why 24?](#)

Learning review

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Possible misconceptions

- Once pupils know how to factorise a quadratic expression of the form $x^2 + bx + c$ they might overcomplicate the simpler case of factorising an expression such as $x^2 + 2x (\equiv (x + 0)(x + 2))$
- Many pupils may think that $(x + a)^2 \equiv x^2 + a^2$
- Some pupils may think that, for example, $-2 \times -3 = -6$
- Some pupils may think that $x^2 + 12 + 7x$ is not equivalent to $x^2 + 7x + 12$, and therefore think that they are wrong if the answer is given as $x^2 + 7x + 12$



KNOWLEDGE

The Big Picture: [Ratio and Proportion progression map](#)

- **solve problems involving direct and inverse proportion including graphical and algebraic representations**
- **apply the concepts of congruence and similarity, including the relationships between lengths in similar figures**
- **change freely between compound units (e.g. density, pressure) in numerical and algebraic contexts**
- **use compound units such as density and pressure**
- **NEW STYLE RATIO QUESTIONS (NEW)**

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SKILLS

- Know the difference between direct and inverse proportion
- Recognise direct and inverse proportion in a situation
- Know the features of a graph that represents a direct and inverse proportion situation
- Know the features of an expression (or formula) that represents a direct and inverse proportion situation
- Understand the connection between the multiplier, the expression and the graph
- Form an equation arising from directly/indirectly proportional relationships
- Find missing values have established an equation describing a direct/indirect proportional relationship
- Know the meaning of congruent shapes
- Know the meaning of similar shapes
- Identify congruence and similarity of shapes in a range of situations
- Identify the information required to solve a problem involving similar shapes
- Finding missing lengths in similar shapes
- Understand why speed, density and pressure are known as compound units
- Know the definition of density (pressure, population density, speed)
- Solve problems involving density (pressure, speed)
- Convert between units of density
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Suggested resources

- KM: [Graphing proportion](#)
 NRICH: [In proportion](#)
 NRICH: [Ratios and dilutions](#)
 NRICH: [Similar rectangles](#)
 NRICH: [Fit for photocopying](#)
 NRICH: [Tennis](#)
 NRICH: [How big?](#)
- Learning review**
- www.diagnosticquestions.com

Prerequisites Mathematical language Agreed Common Teaching Approaches

- Find a relevant multiplier in a situation involving proportion
- Plot the graph of a linear function
- Understand the meaning of a compound unit
- Convert between units of length, capacity, mass and time

Direct proportion
 Inverse proportion
 Multiplier
 Linear
 Congruent, Congruence
 Similar, Similarity
 Compound unit
 Density, Population density
 Pressure

Notation
 Kilograms per metre cubed is written as kg/m^3

Pupils have explored enlargement previously.
 Use the story of Archimedes and his 'eureka moment' when introducing density.
 Up-to-date information about population densities of counties and cities of the UK, and countries of the world, is easily found online.
 NCETM: [The Bar Model](#)
 NCETM: [Multiplicative reasoning](#)
 NCETM: [Departmental workshops: Proportional Reasoning](#)
 NCETM: [Departmental workshops: Congruence and Similarity](#)
 NCETM: [Glossary](#)

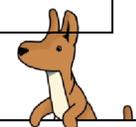
Common approaches
All pupils are taught to set up a 'proportion table' and use it to find the multiplier in situations involving direct proportion

Reasoning opportunities and probing questions Cross Curricular Links Possible misconceptions

- Show me an example of two quantities that will be in direct (inverse) proportion. And another. And another ...
 - Convince me that this information shows a proportional relationship. What type of proportion is it?
- | | |
|----|-----|
| 40 | 3 |
| 60 | 2 |
| 80 | 1.5 |
- Which is the greatest density: 0.65g/cm^3 or 650kg/m^3 ? Convince me.

Cross Curricular Links

- Many pupils will want to identify an additive relationship between two quantities that are in proportion and apply this to solve problems
- The word 'similar' means something much more precise in this context than in other contexts pupils encounter. This can cause confusion.
- Some pupils may think that a multiplier always has to be greater than 1





KNOWLEDGE

The Big Picture: [Algebra progression map](#)

- recognise and use Fibonacci type sequences, quadratic sequences

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SKILLS	Suggested resources	
<ul style="list-style-type: none"> Recognise Fibonacci numbers Recognise the Fibonacci sequence Generate Fibonacci type sequences Find the next three terms in any Fibonacci type sequence Substitute numbers into formulae including terms in x^2 Generate terms of a quadratic sequence Identify quadratic sequences Establish the first and second differences of a quadratic sequence Find the next three terms in any quadratic sequence Find the term in x^2 for a quadratic sequence Compare the term in x^2 and the whole sequence Find the nth term of a sequence of the form $ax^2 + b$ Find the nth term of a sequence of the form $ax^2 + bx + c$ 	<p>KM: Forming Fibonacci equations KM: Mathematician of the Month: Fibonacci KM: Leonardo de Pisa KM: Fibonacci solver. Pupils can be challenged to create one of these. KM: Sequence plotting. A grid for plotting nth term against term. KM: Maths to Infinity: Sequences KM: Stick on the Maths: Quadratic sequences NRICH: Fibs</p> <p>Learning review</p> <ul style="list-style-type: none"> www.diagnosticquestions.com 	
Prerequisites	Mathematical language	Agreed Common Teaching Approaches
<ul style="list-style-type: none"> Generate a linear sequence from its nth term Substitute positive numbers into quadratic expressions Find the nth term for an increasing linear sequence Find the nth term for an decreasing linear sequence 	<p>Term Term-to-term rule Position-to-term rule nth term Generate Linear Quadratic First (second) difference Fibonacci number Fibonacci sequence</p> <p>Notation $T(n)$ is often used to indicate the 'nth term'</p>	<p>The Fibonacci sequence consists of the Fibonacci numbers (1, 1, 2, 3, 5, ...), while a Fibonacci type sequence is any sequence formed by adding the two previous terms to get the next term. NCETM: Departmental workshops: Sequences NCETM: Glossary</p> <p>Common approaches <i>All students should use a spreadsheet to explore aspects of sequences during this unit. For example, this could be using formulae to continue a given sequence, to generate the first few terms of a sequence from an nth term as entered, or to find the missing terms in a Fibonacci sequence as in 'Fibonacci solver'.</i></p>
Reasoning opportunities and probing questions	Cross Curricular Links	Possible misconceptions
<ul style="list-style-type: none"> A sequence has the first two terms 1, 2, ... Show me a way to continue this sequence. And another. And another ... A sequence has nth term $3n^2 + 2n - 4$. Jenny writes down the first three terms as 1, 12, 29. Kenny writes down the first three terms as 1, 36, 83. Who do agree with? Why? What mistake has been made? A sequence starts with the terms 6, 12, 20, 30, ... Find the nth term for this sequence (i.e. $n^2 + 3n + 2$). Look for patterns in how each of the numbers can be constructed. Is there another way to find the nth term (i.e. $(n+1)(n+2)$)? Show that the two nth terms are equivalent. 		<ul style="list-style-type: none"> Some students may think that it is possible to find an nth term for any sequence. A Fibonacci type sequence would require a recurrence relation instead. Some students may think that the second difference (of a quadratic sequence) is equivalent to the coefficient of x^2. Some students may substitute into ax^2 incorrectly, working out $(ax)^2$ instead.



KNOWLEDGE

The Big Picture: [Algebra progression map](#)

- understand and use the concepts and vocabulary of inequalities
- **solve linear inequalities in one variable**
- **represent the solution set to an inequality on a number line**

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SKILLS		Suggested resources
<ul style="list-style-type: none"> • Revision of solving equations with unknowns on one/two sides • Understand the meaning of the four inequality symbols • Choose the correct inequality symbol for a particular situation • Represent practical situations as inequalities • Recognise a simple linear inequality • Find the set of integers that are solutions to an inequality • Use set notation to list a set of integers • Use a formal method to solve an inequality • Use a formal method to solve an inequality with unknowns on both sides • Use a formal method to solve an inequality involving brackets • Know how to deal with negative number terms in an inequality • Know how to show a range of values that solve an inequality on a number line • Know when to use an open circle at the end of a range of values shown on a number line • Know when to use a filled circle at the end of a range of values shown on a number line • Use a number line to find the set of values that are true for two inequalities 		KM: Stick on the Maths: Inequalities KM: Convinced?: Inequalities in one variable NRICH: Inequalities Learning review <ul style="list-style-type: none"> • www.diagnosticquestions.com
Prerequisites	Mathematical language	Agreed Common Teaching Approaches
<ul style="list-style-type: none"> • Understand the meaning of the four inequality symbols • Solve linear equations including those with unknowns on both sides 	(Linear) inequality Unknown Manipulate Solve Solution set Integer Notation The inequality symbols: < (less than), > (greater than), ≤ (less than or equal to), ≥ (more than or equal to) The number line to represent solutions to inequalities. An open circle represents a boundary that is not included. A filled circle represents a boundary that is included. Set notation; e.g. {-2, -1, 0, 1, 2, 3, 4}	The mathematical process of solving a linear inequality is identical to that of solving linear equations. The only exception is knowing how to deal with situations when multiplication or division by a negative number is a possibility. Therefore, take time to ensure pupils understand the concept and vocabulary of inequalities. NCETM: Departmental workshops: Inequalities NCETM: Glossary Common approaches <i>Pupils are taught to manipulate algebraically rather than be taught 'tricks'. For example, in the case of $-2x > 8$, pupils should not be taught to flip the inequality when dividing by -2. They should be taught to add $2x$ to both sides. Many pupils themselves will later generalise.</i>
Reasoning opportunities and probing questions	Cross Curricular Links	Possible misconceptions
<ul style="list-style-type: none"> • Show me an inequality (with unknowns on both sides) with the solution $x \geq 5$. And another. And another ... • Convince me that there are only 5 common integer solutions to the inequalities $4x < 28$ and $2x + 3 \geq 7$. • What is wrong with this statement? How can you correct it? $1 - 5x \geq 8x - 15$ so $1 \geq 3x - 15$. 		<ul style="list-style-type: none"> • Some pupils may think that it is possible to multiply or divide both sides of an inequality by a negative number with no impact on the inequality (e.g. if $-2x > 12$ then $x > -6$) • Some pupils may think that a negative x term can be eliminated by subtracting that term (e.g. if $2 - 3x \geq 5x + 7$, then $2 \geq 2x + 7$) • Some pupils may know that a useful strategy is to multiply out any brackets, but apply incorrect thinking to this process (e.g. if $2(3x - 3) < 4x + 5$, then $6x - 3 < 4x + 5$)



KNOWLEDGE

The Big Picture: [Measurement and mensuration progression map](#)

- **identify and apply circle definitions and properties, including: tangent, arc, sector and segment**
- **calculate arc lengths, angles and areas of sectors of circles**
- **know the formulae for: Pythagoras' theorem, $a^2 + b^2 = c^2$, and apply it to find lengths in right-angled triangles in two dimensional figures (ALSO Stage 8)**
- **know the trigonometric ratios, $\sin\theta = \text{opposite/hypotenuse}$, $\cos\theta = \text{adjacent/hypotenuse}$, $\tan\theta = \text{opposite/adjacent}$ (NEW)**
- **apply it to find angles and lengths in right-angled triangles in two dimensional figures (NEW)**

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SKILLS

- Find the area and circumference of a circle (revision)
- Find the area and perimeter of a quarter or semi-circle (revision)
- Know the vocabulary of circles
- Know how to find arc length
- Calculate the arc length of a sector when radius is given
- Know how to find the area of a sector
- Calculate the area of a sector when radius is given
- Calculate the angle of a sector when the arc length and radius are known
- Calculate exactly with multiples of π
- Know Pythagoras' theorem
- Identify the hypotenuse in a right-angled triangle
- Know when to apply Pythagoras' theorem
- Calculate the hypotenuse of a right-angled triangle using Pythagoras' theorem
- Calculate one of the shorter sides in a right-angled triangle using Pythagoras' theorem
- Use trigonometry in right angled triangles to find a missing side
- Use trigonometry in right angled triangles to find a missing angle
- Use trigonometry in a problem solving context: finding angles and sides.

Suggested resources

- KM: [The language of circles](#)
 KM: [One old Greek](#) (geometrical derivation of Pythagoras' theorem. This is explored further in the next unit)
 KM: [Stick on the Maths: Pythagoras' Theorem](#)
 KM: [Stick on the Maths: Right Prisms](#)
 NRICH: [Curvy Areas](#)
 NRICH: [Changing Areas, Changing Volumes](#)

Learning review

- www.diagnosticquestions.com

Prerequisites Mathematical language Agreed Common Teaching Approaches

- Know and use the number π
- Know and use the formula for area and circumference of a circle
- Know how to use formulae to find the area of rectangles, parallelograms, triangles and trapezia
- Know how to find the area of compound shapes

Circle, Pi
 Radius, diameter, chord, circumference, arc, tangent, sector, segment
 (Right) prism, cylinder
 Cross-section
 Hypotenuse
 Pythagoras' theorem

Notation
 π
 Abbreviations of units in the metric system: km, m, cm, mm, mm², cm², m², km², mm³, cm³, km³

This unit builds on the area and circle work from Stages 7 and 8. Pupils will need to be reminded of the key formula, in particular the importance of the perpendicular height when calculating areas and the correct use of πr^2 . Note: some pupils may only find the area of the three 'distinct' faces when finding surface area.
 Pupils must experience right-angled triangles in different orientations to appreciate the hypotenuse is always opposite the right angle.
 NCETM: [Glossary](#)
Common approaches
Pupils visualize and write down the shapes of all the faces of a prism before calculating the surface area. Every classroom has a set of [area posters](#) on the wall.
Pythagoras' theorem is stated as 'the square of the hypotenuse is equal to the sum of the squares of the other two sides' not just $a^2 + b^2 = c^2$.



Reasoning opportunities and probing questions	Cross Curricular Links	Possible misconceptions
<ul style="list-style-type: none"> • Show me a sector with area 25π. And another. And another ... • Always/ Sometimes/ Never: The value of the volume of a prism is less than the value of the surface area of a prism. • Always/ Sometimes/ Never: If $a^2 + b^2 = c^2$, a triangle with sides a, b and c is right angled. • Kenny thinks it is possible to use Pythagoras' theorem to find the height of isosceles triangles that are not right- angled. Do you agree with Kenny? Explain your answer. • Convince me the hypotenuse can be represented as a horizontal line. 		<ul style="list-style-type: none"> • Some pupils will work out $(\pi \times r)^2$ when finding the area of a circle • Some pupils may use the sloping height when finding cross-sectional areas that are parallelograms, triangles or trapezia • Some pupils may confuse the concepts of surface area and volume • Some pupils may use Pythagoras' theorem as though the missing side is always the hypotenuse • Some pupils may not include the lengths of the radii when calculating the perimeter of an arc



KNOWLEDGE

The Big Picture: [Properties of Shape progression map](#)

- use the basic congruence criteria for triangles (SSS, SAS, ASA, RHS)
- apply angle facts, triangle congruence, similarity and properties of quadrilaterals to conjecture and derive results about angles and sides, including Pythagoras' Theorem and the fact that the base angles of an isosceles triangle are equal, and use known results to obtain simple proofs

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SKILLS		Suggested resources
<ul style="list-style-type: none"> • Know the criteria for triangles to be congruent (SSS, SAS, ASA, RHS) • Identify congruent triangles • Be able to justify congruency using mathematical language referring to known geometrical facts (e.g. opposite sides of a parallelogram are equal or alternate angles are equal) • Use known facts to form conjectures about lines and angles in geometrical situations • Use known facts to derive further information in geometrical situations • Test conjectures using known facts • Know the structure of a simple mathematical proof • Use known facts to create simple proofs • Explain why the base angles in an isosceles triangle must be equal • Explain the connections between Pythagorean triples 	<p>KM: Geometrical proof</p> <p>KM: Shape work: Triangles to thirds, 4x4 square, Squares, Congruent triangles</p> <p>KM: Daniel Gumb's cave</p> <p>KM: Pythagorean triples</p> <p>KM: Stick on the Maths: Congruence and similarity</p> <p>NRICH: Tilted squares</p> <p>NRICH: What's possible?</p> <p>Learning review</p> <ul style="list-style-type: none"> • www.diagnosticquestions.com 	
Prerequisites	Mathematical language	Agreed Common Teaching Approaches
<ul style="list-style-type: none"> • Know angle facts including angles at a point, on a line and in a triangle • Know angle facts involving parallel lines and vertically opposite angles • Know the properties of special quadrilaterals • Know Pythagoras' theorem 	<p>Congruent, congruence</p> <p>Similar (shapes), similarity</p> <p>Hypotenuse</p> <p>Conjecture</p> <p>Derive</p> <p>Prove, proof</p> <p>Counterexample</p> <p>Notation</p> <p>Notation for equal lengths and parallel lines</p> <p>SSS, SAS, ASA, RHS</p> <p>The 'implies that' symbol (\Rightarrow)</p>	<p>'Known facts' should include angle facts, triangle congruence, similarity and properties of quadrilaterals</p> <p>NCETM: Glossary</p> <p>Common approaches</p> <p><i>All students are asked to draw 1, 2, 3 and 4 points on the circumference of a set of circles. In each case, they join each point to every other point and count the number of regions the circle has been divided into. Using the results 1, 2, 4 and 8 they form a conjecture that the sequence is the powers of 2. They test this conjecture for the case of 5 points and find the circle is divided into 16 regions as expected. Is this enough to be convinced? It turns out that it should not be, as 6 points yields either 30 or 31 regions depending on how the points are arranged. This example is used to emphasise the importance and power of mathematical proof. See KM: Geometrical proof</i></p>
Reasoning opportunities and probing questions	Cross Curricular Links	Possible misconceptions
<ul style="list-style-type: none"> • Show me a pair of congruent triangles. And another. And another • Show me a pair of similar triangles. And another. And another • What is the same and what is different: Proof, Conjecture, Justification, Test? • Convince me the base angles of an isosceles triangle are equal. • Show me a Pythagorean Triple. And another. And another. • Convince me a triangle with sides 3, 4, 5 is right-angled but a triangle with sides 4, 5, 6 is not right-angled. 		<ul style="list-style-type: none"> • Some pupils think AAA is a valid criterion for congruent triangles. • Some pupils try and prove a geometrical situation using facts that 'look OK', for example, 'angle ABC looks like a right angle'. • Some pupils do not appreciate that diagrams are often drawn to scale. • Some pupils think that all triangles with sides that are consecutive numbers are right angled.



KNOWLEDGE

The Big Picture: [Algebra progression map](#)

- identify and interpret gradients and intercepts of linear functions algebraically
- use the form $y = mx + c$ to identify parallel lines
- find the equation of the line through two given points, or through one point with a given gradient
- recognise, sketch and interpret graphs of quadratic functions
- recognise, sketch and interpret graphs of simple cubic functions and the reciprocal function $y = 1/x$ with $x \neq 0$
- plot and interpret graphs (including reciprocal graphs) and graphs of non-standard functions in real contexts, to find approximate solutions to problems such as simple kinematic problems involving distance, speed and acceleration

Cross-curricular links

SKILLS	Suggested resources
<ul style="list-style-type: none"> • Use the form $y = mx + c$ to identify parallel lines • Rearrange an equation into the form $y = mx + c$ • Find the equation of a line through one point with a given gradient • Find the equation of a line through two given points • Interpret the gradient of a straight line graph as a rate of change • Interpret the y intercept of a straight line graph in context • Plot graphs of quadratic (cubic, reciprocal) functions • Recognise and interpret the graphs of quadratic (cubic, reciprocal) functions • Sketch graphs of quadratic (cubic, reciprocal) functions • Plot and interpret graphs of non-standard functions in real contexts • Find approximate solutions to kinematic problems involving distance, speed and acceleration 	<p>KM: Screenshot challenge</p> <p>KM: Stick on the Maths: Quadratic and cubic functions</p> <p>KM: Stick on the Maths: Algebraic Graphs</p> <p>NRICH: Diamond Collector</p> <p>NRICH: Fill me up</p> <p>NRICH: What's that graph?</p> <p>NRICH: Speed-time at the Olympics</p> <p>NRICH: Exploring Quadratic Mappings</p> <p>NRICH: Minus One Two Three</p> <p>Learning review</p> <ul style="list-style-type: none"> • www.diagnosticquestions.com

Prerequisites	Mathematical language	Agreed Common Teaching Approaches
<ul style="list-style-type: none"> • Plot straight-line graphs • Interpret gradients and intercepts of linear functions graphically and algebraically • Recognise, sketch and interpret graphs of linear functions • Recognise graphs of simple quadratic functions • Plot and interpret graphs of kinematic problems involving distance and speed 	<p>Function, equation</p> <p>Linear, non-linear</p> <p>Quadratic, cubic, reciprocal</p> <p>Parabola, Asymptote</p> <p>Gradient, y-intercept, x-intercept, root</p> <p>Rate of change</p> <p>Sketch, plot</p> <p>Kinematic</p> <p>Speed, distance, time</p> <p>Acceleration, deceleration</p> <p>Notation</p> <p>$y = mx + c$</p>	<p>This unit builds on the graphs of linear functions and simple quadratic functions work from Stage 8.</p> <p>Where possible, students should be encouraged to plot linear graphs efficiently by using knowledge of the y-intercept and the gradient.</p> <p>NCETM: Glossary</p> <p>Common approaches</p> <p><i>'Monter' and 'commencer' are shared as the reason for 'm' and 'c' in $y = mx + c$ and links to $y = ax + b$</i></p> <p><i>Students plot points with a 'x' and not '•'</i></p> <p><i>Students draw graphs in pencil</i></p> <p><i>All student use dynamic graphing software to explore graphs</i></p>
Reasoning opportunities and probing questions	Cross Curricular Links	Possible misconceptions
<ul style="list-style-type: none"> • Convince me the lines $y = 3 + 2x$, $y - 2x = 7$, $2x + 6 = y$ and $8 + y - 2x = 0$ are parallel to each other. • What is the same and what is different: $y = x$, $y = x^2$, $y = x^3$ and $y = 1/x$? • Show me a sketch of a quadratic (cubic, reciprocal) graph. And another. And another ... • Sketch a distance/time graph of your journey to school. What is the same and what is different with the graph of a classmate? 		<ul style="list-style-type: none"> • Some pupils do not rearrange the equation of a straight line to find the gradient of a straight line. For example, they think that the line $y - 2x = 6$ has a gradient of -2. • Some pupils may think that gradient = (change in x) / (change in y) when trying to equation of a line through two given points. • Some pupils may incorrectly square negative values of x when plotting graphs of quadratic functions. • Some pupils think that the horizontal section of a distance time graph means an object is travelling at constant speed. • Some pupils think that a section of a distance time graph with negative gradient means an object is travelling backwards or downhill.



KNOWLEDGE

The Big Picture: [Algebra progression map](#)

- solve, in simple cases, two linear simultaneous equations in two variables algebraically
- derive an equation (or two simultaneous equations), solve the equation(s) and interpret the solution
- find approximate solutions to simultaneous equations using a graph

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SKILLS		Suggested resources
<ul style="list-style-type: none"> • Understand that there are an infinite number of solutions to the equation $ax + by = c$ ($a \neq 0, b \neq 0$) • Understand the concept of simultaneous equations • Find approximate solutions to simultaneous equations using a graph • Understand the concept of solving simultaneous equations by elimination • Target a variable to eliminate • Decide if multiplication of one equation is required • Decide whether addition or subtraction of equations is required • Add or subtract pairs of equations to eliminate a variable • Find the value of one variable in a pair of simple simultaneous equations • Find the value of the second variable in a pair of simple simultaneous equations • Solve two linear simultaneous equations in two variables in very simple cases (no multiplication required) • Solve two linear simultaneous equations in two variables in simple cases (multiplication of one equation only required) • Derive and solve two simultaneous equations • Interpret the solution to a pair of simultaneous equations 		KM: Stick on the Maths ALG2: Simultaneous linear equations NRICH: What's it worth? NRICH: Warm snug Double Glazing NRICH: Arithmagons Learning review <ul style="list-style-type: none"> • www.diagnosticquestions.com
Prerequisites	Mathematical language	Agreed Common Teaching Approaches
<ul style="list-style-type: none"> • Solve linear equations • Substitute numbers into formulae • Plot graphs of functions of the form $y = mx + c$, $x \pm y = c$ and $ax \pm by = c$ • Manipulate expressions by multiplying by a single term 	Equation Simultaneous equation Variable Manipulate Eliminate Solve Derive Interpret	Pupils will be expected to solve simultaneous equations in more complex cases in Stage 10. This includes involving multiplications of both equations to enable elimination, cases where rearrangement is required first, and the method of substitution. NCETM: Glossary Common approaches <i>Pupils are taught to label the equations (1) and (2), and label the subsequent equation (3)</i> <i>Teachers use graphs (i.e. dynamic software) to demonstrate solutions to simultaneous equations at every opportunity</i>
Reasoning opportunities and probing questions	Cross Curricular Links	Possible misconceptions
<ul style="list-style-type: none"> • Show me a solution to the equation $5a + b = 32$. And another, and another ... • Show me a pair of simultaneous equations with the solution $x = 2$ and $y = -5$. And another, and another ... • Kenny and Jenny are solving the simultaneous equations $x + 4y = 7$ and $x - 2y = 1$. Kenny thinks the equations should be added. Jenny thinks they should be subtracted. Who do you agree with? Explain why. 		<ul style="list-style-type: none"> • Some pupils may think that addition of equations is required when both equations involve a subtraction • Some pupils may not multiply all coefficients, or the constant, when multiplying an equation • Some pupils may think that it is always right to eliminate the first variable • Some pupils may struggle to deal with negative numbers correctly when adding or subtracting the equations



KNOWLEDGE

The Big Picture: [Probability progression map](#)

- **calculate the probability of independent and dependent combined events, including using tree diagrams and other representations, and know the underlying assumptions**
- **enumerate sets and combinations of sets systematically, using tree diagrams**
- **understand that empirical unbiased samples tend towards theoretical probability distributions, with increasing sample size**

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SKILLS		Suggested resources
<ul style="list-style-type: none"> • List outcomes of combined events using a tree diagram • Label a tree diagram with probabilities • Label a tree diagram with probabilities when events are dependent • Know when to add two or more probabilities • Know when to multiply two or more probabilities • Use a tree diagram to calculate probabilities of independent combined events • Use a tree diagram to calculate probabilities of dependent combined events • Understand that relative frequency tends towards theoretical probability as sample size increases • Use a sample space diagram to represent a scenario involving two variables. • Utilise the sample space diagram to find the probability of an event happening. • Represent a scenario using a Venn diagram • Utilise the Venn diagram to find the probability of an event happening • Recognise notation used in Venn diagrams. • Extend Venn diagrams to three separate scenarios (eg study of Biology, History & Maths) 		<p>KM: Stick on the Maths: Tree diagrams KM: Stick on the Maths: Relative frequency KM: The drawing pin experiment</p> <p>Learning review</p> <ul style="list-style-type: none"> • www.diagnosticquestions.com
Prerequisites	Mathematical language	Agreed Common Teaching Approaches
<ul style="list-style-type: none"> • Add fractions (decimals) • Multiply fractions (decimals) • Convert between fractions, decimals and percentages • Use frequency trees to record outcomes of probability experiments • Use experimental and theoretical probability to calculate expected outcomes 	<p>Outcome, equally likely outcomes Event, independent event, dependent event Tree diagrams Theoretical probability Experimental probability Random Bias, unbiased, fair Relative frequency Enumerate Set</p> <p>Notation P(A) for the probability of event A Probabilities are expressed as fractions, decimals or percentage. They should not be expressed as ratios (which represent odds) or as words</p>	<p>Tree diagrams can be introduced as simply an alternative way of listing all outcomes for multiple events. For example, if two coins are flipped, the possible outcomes can be listed (a) systematically, (b) in a two-way table, or (c) in a tree diagram. However, the tree diagram has the advantage that it can be extended to more than two events (e.g. three coins are flipped). NCETM: Glossary</p> <p>Common approaches All students carry out the drawing pin experiment Students are taught not to simply fractions when finding probabilities of combined events using a tree diagram (so that a simple check can be made that the probabilities sum to 1)</p>
Reasoning opportunities and probing questions	Cross Curricular Links	Possible misconceptions
<ul style="list-style-type: none"> • Show me an example of a probability problem that involves adding (multiplying) probabilities • Convince me that there are eight different outcomes when three coins are flipped together • Always / Sometimes / Never: increasing the number of times an experiment is carried out gives an estimated probability that is closer to the theoretical probability 		<ul style="list-style-type: none"> • When constructing a tree diagram for a given situation, some students may struggle to distinguish between how events, and outcomes of those events, are represented • Some students may muddle the conditions for adding and multiplying probabilities • Some students may add the denominators when adding fractions



KNOWLEDGE

- interpret and construct tables, charts and diagrams, including tables and line graphs for time series data and know their appropriate use
- know correlation does not indicate causation; interpolate and extrapolate apparent trends whilst knowing the dangers of so doing
- **construct and interpret diagrams for grouped discrete data and continuous data, i.e. cumulative frequency graphs, and know their appropriate use (NEW)**

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SKILLS		Suggested resources
<ul style="list-style-type: none"> • Construct graphs of time series • Interpret graphs of time series • Construct and interpret compound bar charts • Interpret a wider range of non-standard graphs and charts • Represent data using a stem and leaf graph • Interpret stem and leaf graphs • Understand that correlation does not indicate causation • Interpret a scatter diagram using understanding of correlation • Construct a line of best fit on a scatter diagram • Be able to explain what an outlier is • Be able to describe why extrapolation is flawed. • Use a line of best fit to estimate values • Know when it is appropriate to use a line of best fit to estimate values • Be able to construct a cumulative frequency graph • Be able to interpret a cumulative frequency graph 		<ul style="list-style-type: none"> •
Prerequisites	Mathematical language	Agreed Common Teaching Approaches
<ul style="list-style-type: none"> • Know the meaning of discrete and continuous data • Interpret and construct frequency tables • Construct and interpret pictograms, bar charts, pie charts, tables, vertical line charts, histograms (equal class widths) and scatter diagrams 	Categorical data, Discrete data Continuous data, Grouped data Axis, axes Time series Compound bar chart Scatter graph (scatter diagram, scattergram, scatter plot) Bivariate data (Linear) Correlation Positive correlation, Negative correlation Line of best fit Interpolate Extrapolate Trend Notation Correct use of inequality symbols when labeling groups in a frequency table	Lines of best fit on scatter diagrams are first introduced in Stage 9, although pupils may well have encountered both lines and curves of best fit in science by this time. William Playfair, a Scottish engineer and economist, introduced the line graph for time series data in 1786. NCETM: Glossary Common approaches <i>As a way of recording their thinking, all students construct the appropriate horizontal and vertical line when using a line of best fit to make estimates.</i> <i>In simple cases, students plot the 'mean of x' against the 'mean of y' to help locate a line of best fit.</i>
Reasoning opportunities and probing questions	Cross Curricular Links	Possible misconceptions
<ul style="list-style-type: none"> • Show me a compound bar chart. And another. And another. • What's the same and what's different: correlation, causation? • What's the same and what's different: scatter diagram, time series, line graph, compound bar chart? • Convince me how to construct a line of best fit. • Always/Sometimes/Never: A line of best fit passes through the origin 	KM: Stick on the Maths HD2: Frequency polygons and scatter diagrams Learning review www.diagnosticquestions.com	<ul style="list-style-type: none"> • Some pupils may think that correlation implies causation • Some pupils may think that a line of best fit always has to pass through the origin • Some pupils may misuse the inequality symbols when working with a grouped frequency table

